USE-LEVEL LIFETIME DISTRIBUTION ESTIMATION UNDER DEPENDENT RIGHT CENSORED TEST DATA

Herbert Hove [1]
University of the Witwatersrand
e-mail: *Herbert.Hove@wits.ac.za*

Frank Beichelt
University of the Witwatersrand

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Abstract: Accelerated life testing (ALT) is a practice for estimating unit reliability at normal use conditions using failure data obtained under more severe test conditions. We focus on life tests where a potential critical unit failure at X_2 (unit lifetime) may be avoided by a degraded failure at some random time X_1. Degraded and critical failures are linked through the degradation process, hence the situation under consideration is that of dependent competing risks. We apply the general result that if the copula $C(\cdot,\cdot)$ of (X_1,X_2) is known, competing risks data uniquely determine the marginal distributions at each stress level. Interest here (and in life testing studies in general) is in unit lifetime. Accordingly, our target of estimation is to extrapolate a use-level lifetime distribution from which important reliability measures such as mean lifetime, warranty period among others are derived. The paper is based in part on a PhD thesis by Hove (2014).

1. Introduction

Highly reliable units rarely fail in a test of practical length under normal use conditions. To obtain failure data required for reliability estimation within the allocated time for experiments, units are tested at higher than usual stress conditions, called accelerated life testing (ALT). The way in which data are generally censored in reliability and life testing gives rise to competing risks. Sometimes, a unit may be removed from observation during testing for many different reasons (events). The times at which these different events remove the unit from observation in a life test are modelled by competing risks.

[1] Corresponding author.

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1.1. Motivational modeling framework

Consider a situation where a potential critical unit failure at some random time X_2 may be avoided by other events such as the occurrence of a non-critical failure at some random time X_1. Often X_2 is the minimum of failure times corresponding to failure modes of primary interest whereas X_1 is a censoring time. For a new unit, the first event may either be a critical or non-critical failure. It occurs at time $Z = min(X_1, X_2)$. If $X_1 < X_2$, a non-critical failure is observed and repairing or replacing the unit may act as a right censoring regarding the corresponding critical failure mode. But if $X_2 < X_1$, the unit experiences a critical failure mode. Cooke (1996) developed the concept of random signs censoring, a well-known competing risks model that is tailored for such situations.

We summarise random signs censoring in our notation as follows. Let X_2 be the potential time of occurrence of a critical failure for a unit that is subject to right censoring. During testing, a unit will likely emit some kind of signal (warning of deterioration) such as inferior performance, noise, vibration etc. before failing. Suppose the process of detecting these warnings is independent of the time the unit has been on test. Then the event that the unit's failure is preceded by some other event is independent of the age X_2 at which the unit fails or would have failed if it were not censored. But given that a critical unit failure is censored, the censoring time X_1 may depend on X_2. This is captured in Definition 1:

Definition 1 Let (X_1, X_2) be a pair of life variables. Then the observed variable $Z = min(X_1, X_2)$ along with the identity $J \in (1,2)$ of which is smaller is a random signs censoring of X_2 by X_1 if the event that a critical unit failure is preceded by a censoring variable is independent of X_2.

Motivated by this modeling framework, the present paper applies the random signs censoring model to ALT. Specifically, we consider life tests where unit failure is not a sudden occurrence, but the end point of an underlying degradation process. We further assume that a dominant measurable performance parameter of the unit exists and that its deterioration over time can be associated with unit reliability. That is, the service life of a unit ends around the time the degradation has caused unit performance to reach a specified failure threshold. Consequently, we define unit failure in a life test as some observed level of unit performance.

Consider a light emitting diode (LED) for example. A complete loss of function in a life test (critical failure) may be avoided by the luminosity of a LED falling below an acceptable limit even though it still has some residual functionality (degraded failure). Because we defined unit failure in terms of the observed level of performance, observation in a life test is stopped whenever a unit experiences a degraded or critical failure. As such, a potential critical unit failure at X_2 (unit lifetime) in a life test may be avoided by a degraded failure at some random time X_1.

Our main interest here (and in life testing studies in general) is in the potential time of occurrence of a critical failure X_2, the lifetime of the unit. We therefore regard X_1 as censoring the event of interest. That is, a degraded failure is a signal that a critical failure is likely to follow if the unit is kept on test. If these signals are detected by the testing team, a competing risk X_1 is observed and unit lifetime X_2 is censored.

1.2. The problem

Under the random signs model, we observe $Z = min(X_1, X_2)$ and the identity $J \in (1,2)$ of the risk that achieved the minimum at each stress level. It is well-known (Tsiatis, 1975) that these data contain enough information to estimate subdistribution functions $F^*_{X_1}(t) = P(X_1 \leq t, X_1 < X_2)$ and $F^*_{X_2}(t) = P(X_2 \leq t, X_2 < X_1)$ but not the marginal distributions $F_{X_J}(\cdot)$ unless X_1 and X_2 are stochastically independent or some untestable assumptions about the nature of their stochastic dependence are made. Assuming independence between X_1 and X_2 has no sound physical basis because degraded and critical failures are linked through the degradation process of the unit.

A general way to overcome this problem is to assume a known dependence structure. Stochastic dependence between the censoring variable X_1 and unit lifetime X_2 is the degree to which the occurrence of high (low) values of the one risk variable impacts on the probability of occurrence of values of the other risk variable. This notion of the dependence structure is a matter of relative ranks and is thus completely based on copulas. The copula function is obtained by making marginal probability integral transforms on X_1 and X_2. That is

$$H(t_1, t_2) = P(X_1 \leq t_1, X_2 \leq t_2) = P(U_1 \leq F_{X_1}(t_1), U_2 \leq F_{X_2}(t_2)) = C(F_{X_1}(t_1), F_{X_2}(t_2))$$

where $H(\cdot,\cdot)$ is the joint distribution of (X_1, X_2), $F_{X_J}(\cdot)$, $J = 1,2$ are the marginal distribution functions of X_J and $C(\cdot,\cdot)$ is the copula. Note that the copula $C(\cdot,\cdot)$ is a bivariate distribution function with uniform margins. For a comprehensive treatment of copulas, see Nelsen (1999). We apply the general result of Zheng and Klein (1995) that if the copula $C(\cdot,\cdot)$ of (X_1, X_2) is known, then the competing risks data uniquely determine the marginal distributions $F_{X_J}(\cdot)$. More precisely, assume the underlying copula $C(\cdot,\cdot)$ has continuous second-order partial derivatives with respect to its arguments and that the marginal distributions $F_{X_J}(t)$ exist at each stress level. Then a straightforward calculation (Bunea and Bedford, 2002) yields (1)

$$\begin{aligned} F^*_{X_1}(t) &\equiv P(X_1 \leq t, X_1 < X_2) = \int_0^t \left(\int_{x_1}^{\infty} h(x_1, x_2) dx_2 \right) dx_1 \\ &= F_{X_1}(t) - \int_0^t c_{u_1}(F_{X_1}(x_1), F_{X_2}(x_1)) f_{X_1}(x_1) dx_1 \end{aligned} \tag{1}$$

where $c_{u_1} = \frac{\partial C(u_1,u_2)}{\partial u_1}$ is calculated in $(F_{X_1}(t), F_{X_2}(t))$ and $h(\cdot,\cdot)$ is the joint density of (X_1, X_2). In the same way,

$$F^*_{X_2}(t) = F_{X_2}(t) - \int_0^t c_{u_2}(F_{X_1}(x_2), F_{X_2}(x_2)) f_{X_2}(x_2) dx_2 \tag{2}$$

where $c_{u_2} = \frac{\partial C(u_1,u_2)}{\partial u_2}$ is also calculated in $(F_{X_1}(t), F_{X_2}(t))$. Equations (1) and (2) yield the non-linear system of differential equations

$$\begin{cases} [1 - c_{u_1}(F_{X_1}(t), F_{X_2}(t))] F'_{X_1}(t) = F^{*'}_{X_1}(t) \\ [1 - c_{u_2}(F_{X_1}(t), F_{X_2}(t))] F'_{X_2}(t) = F^{*'}_{X_2}(t) \end{cases} \tag{3}$$

with initial conditions $F_{X_1}(0) = F_{X_2}(0) = 0$ where $f^*_{X_J}(t) = F^{*'}_{X_1}(t)$ are subdensity functions. The marginal distributions $F_{X_J}(\cdot)$, $J = 1,2$ are the solutions of the nonlinear system in (3). Our main interest is in unit lifetime X_2, and hence the identifiability from competing risks data of the lifetime distribution $F_{X_2}(\cdot)$ at all stress levels. We therefore consider a two fold problem in this paper:

(1) We assume that stochastic dependence of X_1 and X_2 at all stress levels is captured by a known copula and that we have a competing risks sample at each stress level. Because the competing risks sample $(min(x_{11},x_{21})),(min(x_{12},x_{22})),...,(min(x_{1n},x_{2n}))$ from $Z = min(X_1,X_2)$ is incomplete, we use expert opinion to estimate the copula. We utilise the observed occurrences of X_1 and X_2 in the competing risks samples to fit derived models of the subdensity functions $f^*_{X_J}(t)$ to test data at each test stress level. The partial derivatives $c_{u_J}(\cdot,\cdot)$, $j = J,2$ of the estimated copula $C(\cdot,\cdot)$ and the fitted subdensity functions $f^*_{X_J}(t) = F^{*'}_{X_1}(t)$ are our inputs in the nonlinear system in (3).

(2) We select an efficient numerical method that solves the nonlinear system in (3) for the marginal distribution functions $F_{X_1}(\cdot)$ and $F_{X_2}(\cdot)$ at each stress level. Noting that our main interest is in unit lifetime X_2, we use the resulting unit lifetime distributions $F_{X_2}(\cdot)$ at the different test stress levels to extrapolate the lifetime distribution of the unit at normal use conditions by applying an ALT procedure.

The use-level lifetime distribution of the unit we obtain by applying an ALT procedure is our ultimate result in this paper. It is from this extrapolated lifetime distribution that important reliability measures of the unit such as mean life, warranty period etc. can be estimated.

1.3. Overview

The remainder of this article is organised as follows. In Section 2, we discuss the selection of the copula model that captures stochastic dependence between the censoring variable X_1 and unit lifetime X_2 at all stress levels. We conclude the section by demonstrating using a simulation study how the assumed copula is estimated from expert opinion if only a competing risk sample is available. The model for subdensity functions $f^*_{X_J}(\cdot)$ is derived in Section 3. In Section 4, we present a numerical example. Test data on unit lifetime and the censoring variable are not readily available. We simulate typical competing risk samples at different test stress levels based on the Class-H insulation data and the estimated copula model. We utilize materials of Section 2 and Section 3 to solve for the marginal distributions $F_{X_2}(\cdot)$ of unit lifetime X_2 (the variable of interest) at the different test stress levels. We conclude the section by applying an ALT procedure to extrapolate the lifetime distribution at normal use conditions from the identified unit lifetime distributions $F_{X_2}(\cdot)$ at the different test stress levels. In Section 5, we study the sensitivity of the extrapolated survival function of the unit to different degrees of stochastic dependence between X_1 and X_2. We conclude the paper in Section 6 by summarising our main results.

2. Copula model selection and estimation

Model choice is a difficult problem with no obvious answer. When modeling the dependence between competing risks (Zheng and Klein, 1995), the important factor is a reasonable estimate of the copula dependence parameter, not the functional form of the copula. See also studies by Escarela and Carriere (2003), Bunea and Mazzuchi (2007) and Kaishev, Dimitrova and Haberman (2007). We follow the same approach in this paper but instead of assuming a known copula with known parameter(s) as in these studies and those by Chen (2010), Lo and Wilke (2010) and Dimitrova, Haberman and Kaishev (2013), we use expert opinion to estimate the copula parameter(s).

We choose a copula class with an interpretation in terms of probabilities of observable quantities that can be assessed by experts in a defensible way. The class of Archimedean copulas has these properties since it has an interpretation in terms of probabilities of observing concordance and discordance pairs. Stochastic dependence between the censoring variable X_1 and unit lifetime X_2 may vary from extreme negative through independence to extreme positive dependence. As such, the chosen copula must capture the full range of dependence. Such copulas are called comprehensive copulas and the only examples in the Archimedean class are the Clayton and the Frank families. The former exhibits greater dependence in the negative tail than in the positive tail whereas the latter is symmetric. There is however no physical justification to suggest asymmetries between the risk variables. For this reason, we choose the Frank copula which was introduced by Genest (1987). It is given by

$$C_{\theta}^{F}(u_1,u_2) = -\frac{1}{\theta}\ln\left[1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{(e^{-\theta}-1)}\right]$$

where $\theta \in (-\infty,+\infty)\backslash\{0\}$ is the copula dependence parameter.

2.1. Frank copula model estimation: Aspects of the problem to elicit

A copula associated with the pair (X_1,X_2) is invariant under strictly increasing transformations of its marginal distributions. Rank correlation exhibits the same scale-invariance property and measures the degree of monotone relationships between variables. The best known rank based measures of dependence are Spearman's ρ and Kendall's τ. In terms of the copula function (Carriere, 1994; Nelsen, 1999) where $I^2 = [0,1]^2$, Spearman's ρ and Kendall's τ are correspondingly given by

$$\rho_{X_1X_2} = 12\int\int_{I^2} C^F(u_1,u_2)du_1du_2 - 3$$

and

$$\tau_{X_1X_2} = 4\int\int_{I^2} C^F(u_1,u_2)dC^F(u_1,u_2) - 1.$$

Parameterisations of copula families by the rank correlation implies that knowledge of the latter identifies the copula. Hence, the rank correlation is the uncertain quantity to be elicited when estimating the chosen Frank copula model. Cooke (1991), Cooke and Goosens (2000) and Bedford and Cooke (2001) all stress the need to elicit on observable quantities as experts are more comfortable answering questions on such quantities. The rank correlation is not an observable quantity. Other quantities are therefore required to indirectly infer the rank correlation. Unlike Spearman's ρ, Kendall's τ has a simple interpretation in terms of probabilities of observing concordance and discordance pairs (Conover, 1999). Hence Kendall's τ (rank correlation henceforth) is the uncertain quantity to be elicited and concordance probability is the assessment variable. The relationship between Kendall's τ and the Frank copula dependence parameter θ, (Escarela and Carriere, 2003) is given by

$$\tau = 1-\frac{4}{\theta}\left(1-\frac{1}{\theta}\int_0^{\theta}\frac{t}{e^t-1}dt\right). \tag{4}$$

2.2. The elicitation process

Elicitation is the process of formulating the beliefs of an expert about an uncertain quantity into a probability distribution for that quantity. Typically, the elicitation process involves the following:

(1) The expert makes specific judgements about the summaries of his or her distribution.

(2) The analyst constructs a fully specified probability distribution from these summaries.

(3) The fitted distribution is checked to see if it adequately represents the expert's beliefs.

We consider elicitation to be a success if the elicited distribution adequately represents the expert's knowledge, regardless of how good that knowledge is. For elicitation approaches regarding dependence, see Clemen, Fischer and Winkler (2000). Instead of choosing experts and obtaining their distributional summaries, we use a simulation study here. Crucially however, all stages of the elicitation process are followed.

2.2.1. Expert elicitation: A simulation study

Denote by $\left(X_1^{(1)}, X_2^{(1)}\right)$ and $\left(X_1^{(2)}, X_2^{(2)}\right)$ two random draws from a population (X_1, X_2) of test units. Label them units 1 and 2 respectively where $X_1^{(1)}$ and $X_1^{(2)}$ are censoring times and $X_2^{(1)}$ and $X_2^{(2)}$ are the respective unit lifetimes. We ask the expert the assessment question:

Suppose it turns out in a life test that unit 2 *has a longer lifetime than unit* 1*, that is* $X_2^{(1)} < X_2^{(2)}$*. What is your probability that a degraded failure for unit* 1 *would also occur before the degraded failure for unit* 2 *in an ALT experiment*?

Our assessment question is asking directly for a concordance probability

$$P\left[\left(X_1^{(1)} - X_1^{(2)}\right)\left(X_2^{(1)} - X_2^{(2)}\right) > 0\right] = p_c.$$

Given the assessed concordance p_c, we obtain Kendall's τ (uncertain quantity) from

$$\tau = 2p_c - 1. \tag{5}$$

Any two independent draws from the population (X_1, X_2) of test units are either concordant or discordant. As such, p_c is the relative frequency for $\left\{X_1^{(1)} < X_1^{(2)} | X_2^{(1)} < X_2^{(2)}\right\}$ when a large sample of pairs of independent draws from a population of test units is observed. It is thus an observable quantity (physically realisable) and its assessment has a natural interpretation in frequency terms. To yield the right data structure, failure times are simulated from a model tailored for situations where the variable of interest is subject to right censoring. One such model is the alert-delay (AD) model of Dijoux and Gaudoin (2009) in (6)

$$X_1 = pX_2 + \xi \tag{6}$$

where unit lifetime X_2 and ξ are independent. We simulate test data (Z, J) at each stress level as follows:

(1) To account for unit degradation, we simulate lifetimes $X_2^{(1)}$ and $X_2^{(2)}$ for the respective units $\left(X_1^{(1)}, X_2^{(1)}\right)$ and $\left(X_1^{(2)}, X_2^{(2)}\right)$ from the Weibull distribution. For simplicity, we simulate the life variable ξ from the exponential distribution.

(2) For a specified value of p, $X_1^{(1)}$ and $X_1^{(2)}$ are obtained from the AD model in (6). To fully exploit the residual life of the unit, we choose a value of p close enough to one.

(3) Count cases where $X_2^{(1)} < X_2^{(2)}$, say m_2. Out of these m_2 cases, count how many are such that $\{X_1^{(1)} < X_1^{(2)}\}$, say m_1. Estimate p_c by $\frac{m_1}{m_2}$ and obtain Kendall's τ from (5).

(4) Repeat k times to obtain $\tau_1, ..., \tau_k$. Use these k simulated Kendall's τ values to estimate the expert's distribution by nonparametric methods such as a kernel density estimate.

Using the `R` code in `Appendix A`, we simulated $k = 1000$ Kendall's τ values. From these, we obtained summaries of the expert's distribution, namely sample `minimum=-0.1467`; `mode=0.1733` and `maximum=0.5733`. The elicited distribution constructed from these summaries and the kernel estimate of the expert's density from the simulated Kendall's τ values are shown in Figure 1 respectively. Subjective distributions are never precise. Hence the elicited distribution is only required to

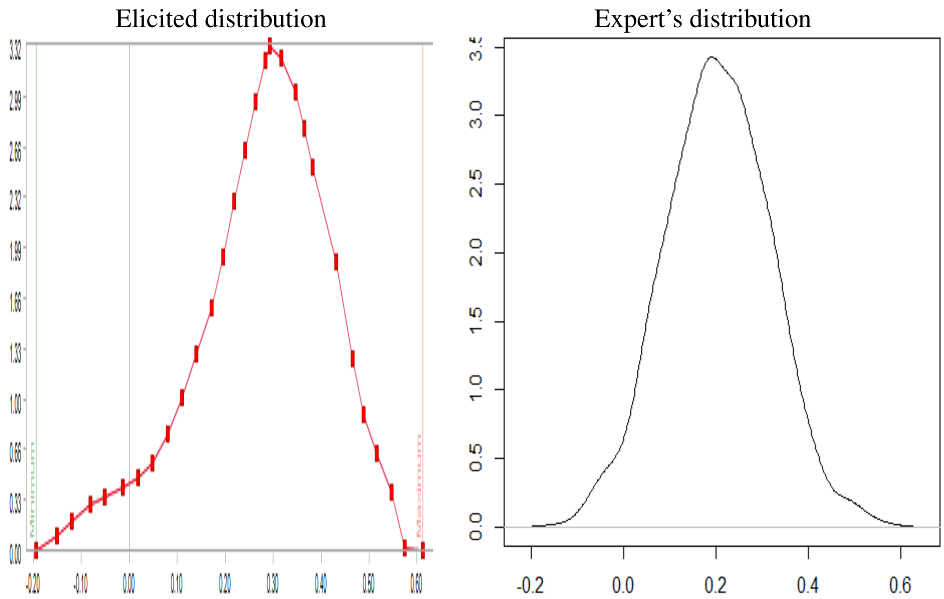


Figure 1: Elicited and expert distribution plots.

closely match the expert's distribution to be practically useful. Main features of the expert's distribution such as peakedness, skewness and spread are clearly captured by the elicited distribution in Figure 1. We therefore consider elicitation to be a success and the parameters of the elicited distribution are reported in Table 1. Our estimate of Kendall's τ (uncertain quantity) is the 50^{th} percentile

Table 1: Parameters of the elicited distribution.

Parameter	Plotted distribution
Mean	0.27816
Standard deviation	0.13823
50^{th} percentile	0.29317

of the elicited distribution. That is $\hat{\tau} = 0.29317$. Substituting $\hat{\tau} = 0.29317$ into (4) yields

$$\frac{4}{\theta}\left(1 - \frac{1}{\theta}\int_0^\theta \frac{t}{e^t - 1}dt\right) = 0.70683. \tag{7}$$

Figure 2 is a plot of the left hand side of (7) from where $\hat{\theta} = 2.8405$. This is our estimate of the Frank copula parameter from expert opinion.

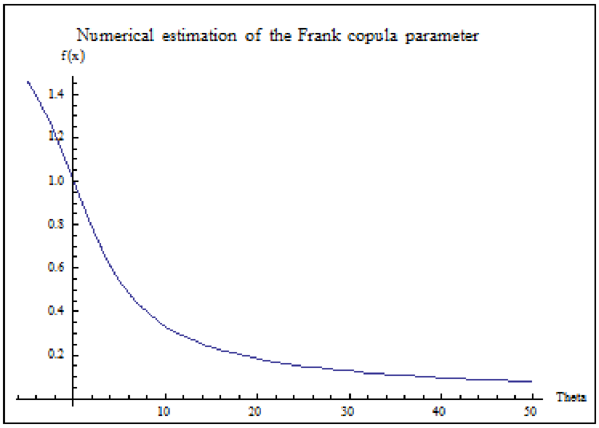


Figure 2: Numerical estimation of the Frank copula parameter.

3. Functional forms (models) for the subdensity functions

We derive in this section functional forms (models) for the subdensity functions $f^*_{X_J}(t) = F^{*'}_{X_1}(t)$ which together with the estimated Frank copula in Section 2 are inputs in the nonlinear system in (3). Recall that unit failure in a life test is defined as the end point of some underlying degradation process. Our estimation of $F^*_J(t)$ at all stress levels is also based on this failure causing process. A degradation process is governed by some random mechanism that can be represented by a stochastic process, say $\{X(t), t \in \mathbb{R}^+\}$. In applications, $\{X(t), t \in \mathbb{R}^+\}$ is generally required to have continuous sample paths. This is in part, why the simple Wiener process is the basic degradation model. For applications where the degradation process is restricted to functions with monotone sample paths, the gamma process is often preferred.

We follow Singpurwalla's (2006b) conceptualisation of degradation as an unobservable construct that is viewed as causing observable surrogates such as unit performance. Hence we consider the degradation process to be a bivariate stochastic process $\{M(t), R(t), t \in \mathbb{R}^+\}$, with $\{M(t), t \in \mathbb{R}^+\}$ the unobservable degradation process and $\{R(t), t \in \mathbb{R}^+\}$ the observable marker process. There is considerable research focusing on the probability structure of the bivariate stochastic process $\{M(t), R(t), t \in \mathbb{R}^+\}$. Cinlar (1972) postulated a Markov additive process but this process still has no developed statistical theory. Whitmore, Crowder and Lawless (1998) assumed a bivariate Wiener process. Its drawback is that $\{M(t), t \in \mathbb{R}^+\}$ is no longer non-decreasing in t when taken to be a Wiener process and no probabilistic link exists between the two.

The probability structure, and one we adopt in this paper is motivated by (Singpurwalla, 2006a; Singpurwalla, 2006b) as follows. Consider the survival function of T, that is $P(T \geq t)$, $t \geq 0$ and let $h(t)$ be its hazard rate function. Denote by $H(t) = \int_0^t h(u)du$, the cumulative hazard at t. By the exponentiation formula with $H(t)$ is specified (Barlow and Proscham, 1975),

$$P(T \geq t; H(t), t \geq 0) = e^{-H(t)}. \tag{8}$$

The right hand side of (8) is the survival function of some random variable, say S, distributed exponentially with scale parameter $\lambda = 1$ and evaluated at $H(t)$. That is

$$P(T \geq t; H(t), t \geq 0) = e^{-H(t)} = P(S \geq H(t)|\lambda = 1). \tag{9}$$

By (9), a test unit fails when its cumulative hazard $H(t)$ first crosses a random threshold S distributed as exponential with scale parameter $\lambda = 1$. A contraction of the clock time from t to $H(t)$ signals acceleration in a life test. Singpurwalla (2006b) describes the marker process $\{R(t), t \in \mathbb{R}^+\}$ by a Wiener process $\{W(t), t \in \mathbb{R}^+\}$ and the latent failure causing process $\{H(t), t \in \mathbb{R}^+\}$ by the Wiener maximum process $W(t)^+ = \left\{\sup_{0 \leq u \leq t} W(u), u \geq 0\right\}$. This probability structure has two advantages. First, the link between the marker and the unobservable processes is obvious from

$$W(t)^+ = \left\{\max_{0 \leq u \leq t} W(u), u \geq 0\right\}. \tag{10}$$

Second, both $\{W(t), t \in \mathbb{R}^+\}$ and $\left\{\sup_{0 \leq u \leq t} W(u), u \geq 0\right\}$ have continuous sample paths and the latter is non-decreasing in t as required. Our point of departure from this derivation is that we assume a fixed failure threshold as follows. Adequate unit performance is specified by industrial standards and a unit fails whenever performance no longer conforms to set standards. Hence it is reasonable to assume a deterministic failure threshold, otherwise failure during testing will not be well defined in our formulation.

Thus a test unit experiences the Wiener maximum process during testing and its failure time is the first passage time of $\{W^+(t), t \geq 0\}$ to the deterministic failure threshold. Denoted by T_s the failure time of the unit. Then

$$T_s = min\left\{u \in \mathbb{R}^+ : \max_{0 \leq u \leq t} W(u) \geq s\right\}$$

and no additional degradation data are necessarily required when deriving the model for subdensity functions $f^*_{X_J}(t)$. As a consequence of (10), the first passage time of $\{W^+(t), t \geq 0\}$ to a failure threshold coincides with the first crossing time of $\{W(t), t \geq 0\}$ to the same failure threshold. The latter is well-known (Chhikara and Folks, 1989) to be inverse Gaussian with mean μ and scale λ. Consequently, $f^*_{X_j}(t)$, $j \in (1,2)$ are distributed as inverse Gaussian with density

$$f^*_{X_J}(t;\mu,\lambda) = \begin{cases} \sqrt{\frac{\lambda}{2\pi t^3}}\exp\left(-\frac{\lambda(t-\mu)^2}{2\mu^2 t}\right); & t > 0,\ j \in (1,2) \\ 0; & \text{otherwise} \end{cases}$$

since X_1 and X_2 have the interpretation of first passage times to a failure threshold. Statistical inference when the failure threshold is assumed to be deterministic is presented in `Appendix B`.

4. Numerical example

4.1. Test data

We assume in this paper that we have a competing risk sample $Z = min(X_1, X_2)$ along with the cause $J \in (1,2)$ that achieved the minimum at each of the k test stress levels. Let temperature be the accelerating variable and that $n_1, n_2, ..., n_k$ units are tested at $T_1{}^\circ C, T_2^\circ C, ..., T_k^\circ C$ stress level respectively. The general structure of competing risks test data is given in Table 2. Observe that

Table 2: General structure of competing risks test data.

$T_1^\circ C$	Z	J	$T_2{}^\circ C$	Z	J	.	.	.	$T_k^\circ C$	Z	J
1	z_1	1	1	z_1	2	.	.	.	1	z_1	2
2	z_2	2	2	z_2	1	.	.	.	2	z_2	1
3	z_3	2	3	z_3	2	.	.	.	3	z_3	1
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
n_1	z_{n_1}	1	n_2	z_{n_2}	2	.	.	.	n_k	z_{n_k}	1

if $J = 1$, the observed value of Z is the time of occurrence of a graded failure. If $J = 2$, the observed value of Z is the time of occurrence of a critical failure. Test data of the form in Table 2 contain enough information to estimate subdistribution functions $F^*_{X_1}(t) = P(X_1 \leq t, X_1 < X_2)$ and $F^*_{X_2}(t) = P(X_2 \leq t, X_2 < X_1)$ at each stress level. These data are however not enough to estimate the distribution functions $F_{X_1}(t) = P(X_1 \leq t)$ and $F_{X_2}(t) = P(X_2 \leq t)$ which are of interest unless additional assumption on the relationship between X_1 and X_2 are imposed. We are particularly interested in $F_{X_2}(\cdot)$, the lifetime distribution of the unit.

Test data are confidential (commercially sensitive) and are generally difficult to access. We do not have test data where the potential time of occurrence of a critical failure is subject to right censoring as in Table 2. Our analysis relies on typical competing risk samples at different test stress levels that we simulate based on the Class-H insulation data (Nelson, 2004) and the estimated Frank copula model in Section 2. The Class-H insulation data are from a temperature-accelerated life test of motorettes insulation that yielded three insulation failure modes; Turn, Phase and Ground failures.

Bunea and Mazzuchi (2007) used the same data set and grouped the data into two competing risk classes: Risk 1 (Turn failure mode) and Risk 2 (Phase and Ground failure modes). We group the data the same way and consider the time of occurrence of Risk 2 as the minimum of the time of occurrence of Phase and Ground failure modes.

An analysis of the Class-H insulation data (Nelson, 2004) found Turn failure to be the earliest failure mode at the design temperature of $180°C$. The motorette was subsequently redesigned to eliminate Turn failure mode. Hence the lifetime of the redesigned motorette is the minimum of Phase and Ground failure modes. We consider Risk 2 analogous to critical failure mode because its occurrence in a life test ends the useful life of the redesigned motorette. That is, Risk 2 is the failure mode of interest. Risk 1 is considered analogous to degraded failure mode because its occurrence would censor the failure mode of interest. Table 3 contains test data that we derived from the Class-H insulation data. These data are not in the required general structure given in Table 2 because of

Table 3: Derived competing risks data at the different test stress levels.

$190°C$	X_1	X_2	$220°C$	X_1	X_2	$240°C$	X_1	X_2	$260°C$	X_1	X_2
1	7228	10511	1	1764	2436	1	1175	1175	1	1632+	600
2	7228	11855	2	2436	2436	2	1881+	1175	2	1632+	744
3	7228	11855	3	2436	2436	3	1521	1881+	3	1632+	744
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
10	10511	12191+	10	3108	4116+	10	1953	1953+	10	1896	1896

the test design which guarantees complete random samples $(x_{11},x_{21}),(x_{12},x_{22}),...,(x_{1n},x_{2n})$ from (X_1,X_2) at all stress levels. Each of Turn, Phase and Ground failure modes occurred on a separate part of the motorette. The first occurring failure mode was isolated while the unit was kept on test until the second or third failure mode occurred. Hence Table 3 contains pseudo-competing risks data with enough information to estimate the marginal distribution functions $F_{X_1}(t) = P(X_1 \leq t)$ and $F_{X_2}(t) = P(X_2 \leq t)$ at each test stress level. Accordingly, an ALT procedure can be applied to extrapolate a use-level lifetime distribution from test data in Table 3 in a straightforward way.

In a typical competing risks situation however, the first occurring failure mode terminates observation in a life test and incomplete random samples $(min(x_{11},x_{21})),(min(x_{12},x_{22})),...,(min(x_{1n},x_{2n}))$ from $Z = min(X_1,X_2)$ are observed at all stress levels. We use our estimated Frank copula in Section 2 and test data in Table 3 to simulate typical competing risks samples that are in the required general structure in Table 2 as follows:

(1) Fit life distributions to the derived competing risks data in Table 3 at each test stress level.

(2) Generate bivariate outcomes (X_1,X_2) from the estimated Frank copula model in Section 2 using the fitted life distributions from the first step when inverting. Obtain $Z = min(X_1,X_2)$ together with the identity $J \in (1,2)$ of the mode that achieves the minimum.

We found the Weibull distribution with scale parameter α and shape parameter β to adequately describe test data in Table 3 for each failure mode and at each test stress level (Hove, 2014). Assuming Weibull marginals (Genest, 1987), we generate observed occurrences of degraded and critical failure times in a competing risks framework at each stress level using the following algorithm:

Algorithm 4.1: **Generating degraded and critical failure data using Frank's copula**

1. Generate independent uniform (0,1) random variables U_1 and U_2.

2. Set $X_1 = F_1^{-1}(U_1) = \alpha_1 \left(ln \frac{1}{1-U_1} \right)^{1/\beta_1}$ where α_1 and β_1 are the ML estimates of the Weibull scale and shape parameters for the degraded failure mode at a stress level.

3. Calculate X_2 as the solution to the equation

$$U_2 = e^{-\theta U_1} \left[\frac{e^{-\theta F_2(X_2)} - 1}{e^{-\theta} - 1 + (e^{-\theta U_1} - 1)(e^{-\theta F_2(X_2)} - 1)} \right].$$

   That is calculate $X_2 = F_2^{-1}(U_{*2}) = \alpha_2 \left(ln \frac{1}{1-U_{*2}} \right)^{1/\beta_2}$ where $U_{*2} = -\frac{1}{\theta} ln \left[\frac{U_2 e^{-\theta} + e^{-\theta U_1}(1-U_2)}{U_2 + e^{-\theta U_1}(1-U_2)} \right]$. The parameters α_2 and β_2 are the ML estimates of the Weibull scale and shape parameters for the corresponding critical failure mode at a stress level. The parameter θ is the Frank copula parameter estimated from expert opinion.

4. Obtain $Z = min(X_1, X_2)$ and the identity of the mode that achieved the minimum.

This algorithm yields test data in Table 4:

Table 4: Simulated test data on unit lifetime under dependent right censoring.

190°C	Z	J	220°C	Z	J	240°C	Z	J	260°C	Z	J
1	8322	1	1	2362	2	1	915	2	1	1399	2
2	9491	1	2	2663	1	2	11751	2	2	1264	2
3	6329	1	3	1728	1	3	1548	1	3	1274	2
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
20	9814	1	20	2801	1	20	1666	1	20	935	2

where column J indicates the identity of the mode that achieved the minimum. These data are in the required general structure given in Table 2.

4.2. Identifiability of the marginal distributions

Our analysis assumes that we have a competing risks sample in hand at each stress level as shown in Table 4. We now apply the general result of Zheng and Klein (1995) that if the copula $C(\cdot,\cdot)$ of (X_1, X_2) is known, competing risks data in Table 4 uniquely determine the marginal distributions $F_{X_J}(\cdot)$. For test data in Table 4, we know the copula since they are simulated from the Frank copula that we estimated using expert opinion in Section 2. The observed occurrences in Table 4 of X_1 (observed when $J = 1$) and X_2 (observed when $J = 2$) allow one to estimate subdensity functions $f^*_{X_J}(t) = F^{*'}_{X_1}(t)$. These are postulated to be inverse Gaussian in Section 3. Estimates for the inverse

Table 5: Inverse Gaussian parameter estimates for the simulated test data.

$190°C$	X_1	X_2	$220°C$	X_1	X_2
μ	8835.214	7578.5	μ	2694.417	2033
λ	589669.5	402153.6	λ	87582.48	17499.47
$240°C$	X_1	X_2	$260°C$	X_1	X_2
μ	1682.429	3061.833	μ	*NA*	1187.421
λ	80450.86	2744.643	λ	*NA*	4519.269

Gaussian scale parameter λ and mean μ for test data in Table 4 are reported in Table 5. No parameter estimates are reported for the censoring variable at $260°C$ stress level because the event $\{X_1 < X_2\}$ was observed only once with the rest corresponding to the event $\{X_2 < X_1\}$. Simulated test data at the $260°C$ stress level are dropped in further analysis as we could not fit the assumed inverse Gaussian distribution to a single data point. The partial derivatives $c_{u_J}(\cdot,\cdot)$ of the estimated Frank copula $C(\cdot,\cdot)$ in Section 2 and estimates of the subdensity functions $f^*_{X_J}(t) = F^{*'}_{X_1}(t)$ in Table 5 are our inputs in the nonlinear system in (3). Figure 3 shows our numerical solutions of the marginal survival functions of unit lifetime X_2 at the different stress levels. We used the Mathematica built-in function `NDSolve` to numerically solve the nonlinear system in (3) for the marginal distributions $F_{X_J}(\cdot)$ reported in Figure 3. For a similar application in biostatistics, see Kaishev et al. (2007) and Dimitrova et al. (2013) for example. Numerical solutions of the marginal survival functions of the censoring variable X_1 are not reported at all stress levels because it is not the variable of interest.

4.3. ALT procedure: Extrapolating the use-level lifetime distribution

Having identified the marginal survival functions of unit lifetime X_2 at the different stress levels in Section 4.2, we apply an ALT procedure to extrapolate the lifetime distribution of the unit at the use-level temperature of $180°C$. We assume the Arrhenius life-stress relationship because it is derived for temperature dependence and is well-known (Nelson, 2004) to be a valid model in insulation work. Under the assumed Arrhenius model, we found the scatter in unit lifetime to be adequately described by a Weibull distribution. This suggests that the Arrhenius-Weibull model is appropriate ALT model. It assumes that

(1) The Weibull distribution adequately describes the scatter in unit lifetime data at each stress level.

(2) The Weibull shape parameter β does not change with stress and the relationship between the Weibull scale parameter α (quantifiable life measure) and temperature (stress) is linear.

We used both graphical and analytical methods to check the adequacy of the Arrhenius-Weibull model. The life-stress relationship and the extrapolated survival function of the unit at the use-level temperature of $180°C$ are displayed in Figure 4. The estimated Weibull distribution parameters at the use-level temperature of $180°C$ are $\hat{\alpha} = 13876$Hr and $\hat{\beta} = 4.411$ where the former estimates the lifetime of the redesigned unit. Selected reliability measures calculated from the extrapolated

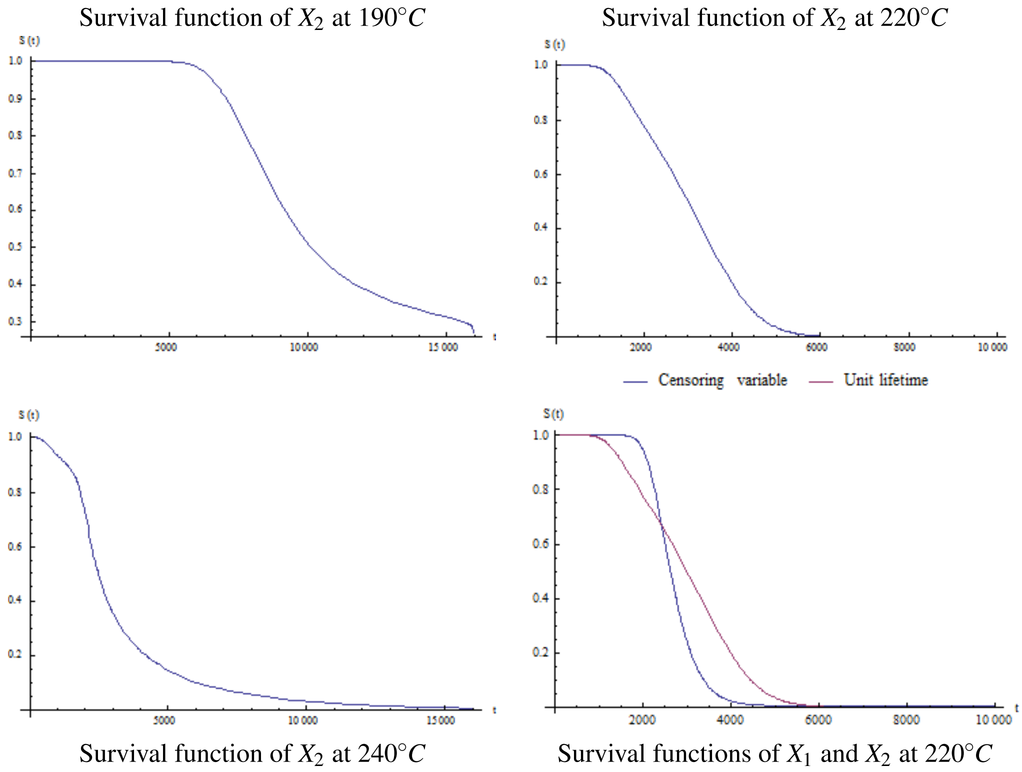


Figure 3: Numerical solutions of survival functions of unit lifetime at different stress level. The bottom right plot shows survival functions of both the censoring variable and unit lifetime.

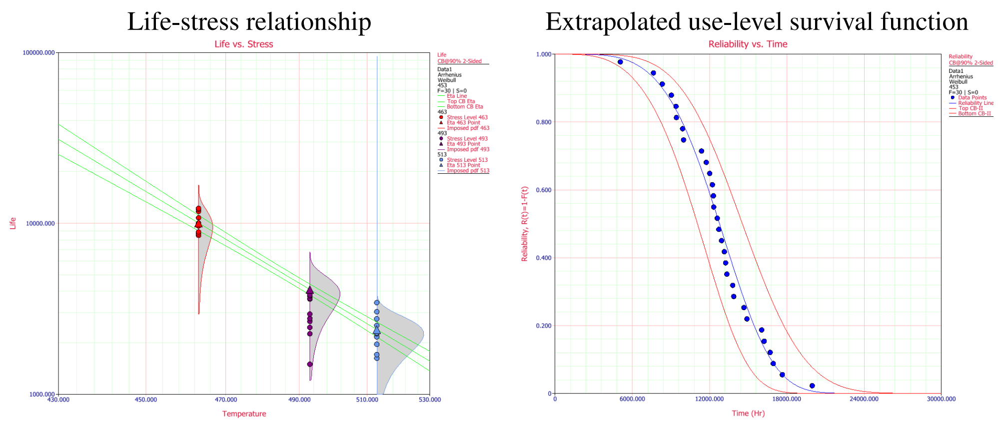


Figure 4: Life-stress relationship and use-level survival function plots.

use-level survival function are given in Table 6 where $B50\%$ life is the time by which 50% of the

Table 6: Selected reliability measures at use-level temperature.

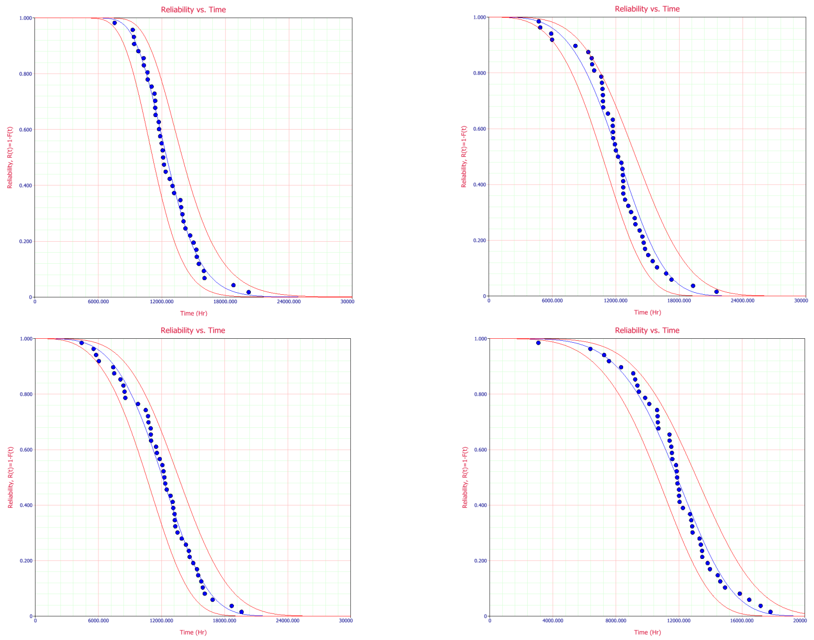
	Reliability measure	90% Confidence limits
$B50\%$ life	12770Hr	$(11178, 14590)$
Mean life	12649Hr	$(11066, 14458)$
Warranty time	7077Hr	$(5736, 8732)$

units in a population will have failed.

5. Sensitivity analysis

The adopted copula-based competing risks methodology largely depends on the elicited rank correlation and hence, the estimated copula model parameter. We present in Figure 5 the sensitivity of

Survival function: $\tau = 0.25$, $\theta = 2.371$ Survival function: $\tau = 0.5$, $\theta = 5.736$



Survival function: $\tau = 0.75$, $\theta = 14.138$ Survival function: $\tau = 0.9$, $\theta = 38.281$

Figure 5: Use-level survival functions of the unit assuming different degrees of dependence.

the extrapolated use-level survival function of the unit with respect to different degrees of stochastic dependence between the risk variables. These survival functions are obtained by solving the non-linear system in (3) for values of θ that correspond to Kendall's τ values equal to 0.25, 0.5, 0.75 and 0.9. The extrapolated use-level survival functions in Figure 5 reveal an apparent shift to the left as the stochastic dependence between the risks increases. That is, as the strength of the rank correlation increases, there is poor survival with respect to the remaining failure mode. This is made clearer by looking at estimated reliability measures in Table 7. Assuming strong stochastic dependence between the censoring variable X_1 and unit lifetime X_2, the latter will continue to operate in the same way following the removal of the former. As a result, cause removal will not significantly improve survival with respect to the remaining modes. However, our results in Table 7 show slight differences in selected reliability measures for different rank correlation values. In particular, *B*50% Life is approximately 12000Hr for the considered four rank correlation values.

Table 7: Sensitivity of estimated reliability measures to different degrees of dependence.

	$\tau = 0.25$	$\tau = 0.5$	$\tau = 0.75$	$\tau = 0.9$
*B*50% Life	12456Hr	12270Hr	12092Hr	11881Hr
Mean life	12704Hr	12190Hr	12011Hr	11745Hr

This somewhat surprising result of slight differences in reliability measures for different degrees of dependence was also obtained by Meeker, Escober and Hong (2009). In their analysis however, they assumed a bivariate lognormal model for the competing risks whereas our investigation assumes a copula model. The practical implication of our result is the conclusion that when estimating the marginal survival functions from dependent competing risk data, one may use a degree of dependence believed to be realistic to admit for the application under consideration.

6. Concluding remarks

We have demonstrated in this paper how the knowledge of the copula identifies the marginal distributions if all one has is a competing risks sample. The paper extends the earlier work of:

(1) Zheng and Klein (1995) and Bunea and Mazzuchi (2007) by answering the question of how to estimate the rank correlation between competing causes of failure. We used expert opinion to estimate the rank correlation, and hence the assumed copula model. Our quantitative method for modeling expert opinion may be applied to other application areas.

(2) Kaishev et al. (2007) and Dimitrova et al. (2013) by deriving the model for the subdistribution functions from the degradation process of the unit.

We further extended the general result of Zheng and Klein (1995) to accelerated testing by extrapolating a use-level lifetime distribution from the survival functions of unit lifetime at the different stress levels. Our study of the sensitivity of the extrapolated use-level lifetime distribution to different degrees of stochastic dependence reveal poor survival with respect to the remaining failure mode (unit lifetime) as the strength of rank correlation increases. We conclude that assuming strong stochastic dependence between the risks, cause removal will not significantly improve survival with respect to the remaining failure mode since it will continue to operate in the same way as the removed mode (censoring variable).

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Appendix A

The R code for assessing p_c and generating an estimated value of Kendall's τ.

```
simwei=function(n,a,b,p,e){
x2=rweibull(n,shape=a,scale=b)
y2=rep(0,n)
z1=rexp(n,e)
z2=rexp(n,e)
count=1
repeat{
if(count==(n+1)) break
y=rweibull(1,shape=a,scale=b)
if(y>x2[count]){
y2[count]=y
count=count+1
}
}
x1=p*x2+z1
y1=p*y2+z2
k=0
for(i in 1:n){
if(y1[i]>x1[i]) k=k+1
}
prob=k/n
tau=2*prob-1
list(x2=x2,y2=y2,z1=z1,z2=z2,x1=x1,y1=y1,n=n,k=k,prob=prob, tau=tau)
}
sim1=simwei(n=75,a=3,b=1,p=0.85,e=1)
simN=1000
output=c(0,0,0)
for(s in 1:simN){
out=simwei(n=75,a=3,b=1,p=0.85,e=1)
output=rbind(output,c(out$n,out$k,out$prob,out$tau))
}
output=output[-1,]
colnames(output)=c("n","k","prob","tau")
output=as.data.frame(output)
x=output$tau
min(x)
names(sort(-table(x)))[1]
max(x)
plot(density(x),main="Kernel density estimate",xlab="Kendall's tau")
```

Appendix B

Statistical inference when the failure threshold is assumed known.

Let $\theta = (\mu, \lambda)^T$ be a vector of the inverse Gaussian distribution parameters. Assuming $t_1, t_2, ..., t_n$ are inverse Gaussian distributed test data for the j^{th} failure mode at a stress level, the log-likelihood function is given by

$$\ell_n(\theta | t_1, ..., t_n) = \frac{n}{2} ln\lambda - \frac{n}{2} ln(2\pi) - \frac{3}{2} \sum_{i=1}^{n} ln(t_i) - \frac{\lambda}{2\mu^2} \sum_{i=1}^{n} \frac{(t_i - \mu)^2}{t_i}.$$

Maximum likelihood (ML) estimates of μ and λ are well-known (Chhikara and Folks, 1989) to be given by

$$\hat{\mu} = \overline{T} = \frac{1}{n} \sum_{i=1}^{n} T_i \qquad \text{and} \qquad \hat{\lambda} = \frac{n}{\sum_{i=1}^{n} \left(\frac{1}{T_i} - \frac{1}{\overline{T}} \right)},$$

respectively. In a competing risk situation however, the contribution to the likelihood function when unit lifetime is censored during testing is the subdensity function of X_1. It is given by

$$f_1^*(x_1; \mu_1, \lambda_1) = q \sqrt{\frac{\lambda_1}{2\pi x_1^3}} \exp\left(- \frac{\lambda_1 (x_1 - \mu_1)^2}{2\mu_1^2 x_1} \right).$$

Similarly, the contribution to the likelihood function when a the occurrence of a critical failure removed a unit from observation during testing is the subdensity function of X_2 given by

$$f_2^*(x_2; \mu_2, \lambda_2) = (1-q) \sqrt{\frac{\lambda_2}{2\pi x_2^3}} \exp\left(- \frac{\lambda_2 (x_2 - \mu_2)^2}{2\mu_2^2 x_2} \right).$$

Lindqvist and Skogsrud (2009) gives a different parameterisation of the inverse Gaussian distribution. Let $(z_1, ..., z_N) = (x_{11}, ..., x_{1n}; x_{21}, ..., x_{2m})$ be the observed competing risks data at each test stress level. Then the likelihood function is given by

$$\begin{aligned} L &= \prod_{i=1}^{n} f_1^*(x_{1i}) \prod_{k=1}^{m} f_2^*(x_{2k}) \\ &= q^n (1-q)^m \frac{\lambda_1^n \lambda_2^m}{2\pi^{\frac{n+m}{2}}} \left(\prod_{i=1}^{n} x_{1i} \right)^{-3/2} \left(\prod_{k=1}^{m} x_{2k} \right)^{-3/2} \\ &\quad \times \exp\left(- \sum_{i=1}^{n} \frac{\lambda_1 (x_{1i} - \mu_1)^2}{2\mu_1^2 x_{1i}} - \sum_{k=1}^{m} \frac{\lambda_2 (x_{2k} - \mu_2)^2}{2\mu_2^2 x_{2k}} \right). \end{aligned}$$

Thus ML estimates of model parameters are obtained by calculating the loglikelihood function, taking partial derivatives with respect to the parameter and solving the resulting likelihood equations. For example, the likelihood equation for the probability q of observing a degraded failure in a life test before the unit reaches the end of its useful life is $n(1-q) - mq = 0$. Hence the ML estimate $\hat{q}$ of q is given by $\hat{q} = \frac{n}{n+m}$ while in practice, readily available optimisation software are used to obtain parameter estimates.

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