EVALUATING RISK IN PRECIOUS METAL PRICES WITH GENERALISED LAMBDA, GENERALISED PARETO AND GENERALISED EXTREME VALUE DISTRIBUTIONS

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Abstract: In this study we investigate the performance of the generalised lambda distribution (GLD), the generalised Pareto distribution (GPD) and the generalised extreme value distribution (GEVD) in modelling daily platinum, gold and silver price log-returns. Our primary goal is to compare GLD against GPD, and GEVD, in the estimation of Value-at-Risk (VaR) and expected shortfall (ES) as per the international Basel regulatory framework. Our analyses show that GPD and GLD generally outperform GEVD for VaR and ES estimation for negative precious metal returns. For gold, the GPD stands out as the most suitable model. For platinum, GPD and GLD are equally adequate, especially at the 1% VaR level. For silver, GLD is the most suitable at 1% VaR level, whereas GPD is the best model at 0.1%. This study has shown that GLD is a suitable model for extreme risk in precious metal prices and can be used for the estimation of VaR and ES values.

1. Introduction

The prices of precious metals, such as gold, silver and platinum, fluctuate substantially over time and this introduces risk on its own (Ren and Giles, 2010). Extreme value theory (EVT) is used to model the behaviour of extreme observations in a series (the tails of the distribution). Classical results in EVT characterises extremes in a data series using two methods, namely the block maxima

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method and the peaks-over-threshold method (POT), and identifies their limiting behaviour with the generalised extreme value distribution (GEVD) and the generalised Pareto distribution (GPD), respectively. In our study, we use the GPD, the GEVD and, in addition, the generalised lambda distribution (GLD) to model extreme daily platinum, gold, and silver price log-returns.

There exists a large literature on using EVT for the estimation of risk measures in areas where extreme observations are of interest, such as finance, insurance, hydrology, climatology and engineering. Specifically, numerous studies in finance and commodity markets have been conducted using EVT, including Embrechts, Klüppelberg and Mikosh (1997), Gençay and Selçuk (2004) and Gilli and Këllezi (2006). Byström (2005) applied EVT to the case of large electricity prices and declared an adequate fit was observed for GPD. Bali (2003) examined the different types of asymptotic distributions for modelling extreme changes in US treasury yields. He found that the thin-tailed Gumbel and exponential distributions performed worse than the fat-tailed Fréchet and Pareto distributions. Marohn (2005) studied the tail index in the case of generalised order statistics and determined the asymptotic properties of the Fréchet distribution.

The GLD has been used in a different context in finance. Lee (2003) showed that GLD is adequate for modelling spot exchange rates. Chalabi, Scott and Wurtz (2010) proposed GLD as an alternative to stable distributions and the Student's t-distribution in modelling equities from NASDAQ-100 index. Beena and Kumaran (2010) modelled the personal income data of a population with GLD. Corrado (2001) used GLD to model security price distributions. Other notable studies on the theory and applications of the GLD were undertaken by Su (2007), Corlu and Corlu (2015) and Corlu, Meterelliyoz and Tiniç (2016). However, to the best of our knowledge, there is limited discussion on fitting daily precious metal returns with GLD and comparing its performance against EVT models (i.e., GPD and GEVD). Vee, Gonpot and Sookia (2012) also showed that different financial indices may be best depicted by different distributions, indicating the need for the study herewith.

In this paper we extend the work of Corlu and Corlu (2015) and Chen and Giles (2014). Specifically, we investigate the performances of the GLD, the GPD and the GEVD in modelling extreme risks for daily platinum, gold and silver log-returns. This is done by utilising graphical analyses (such as excess distribution plots, plot of the tail of underlying distribution and scatter plot of residuals) and various backtesting procedures (i.e., Kupiec test, Chritoffersen test and bootstrap test) to draw robust conclusions on the adequacy of GLD, GPD and GEVD models for Value-at-Risk (VaR) and expected shortfall (ES) estimates.

The rest of the paper is organised as follows. In Section 2 we present a short review on the precious metals market. Section 3 describes the distributions used in the study; GLD, GPD and GEVD. Section 4 introduces risk measures, and their corresponding backtesting procedures, that are utilised for this study. Empirical results from GLD, GPD and GEVD estimation are discussed in Section 5. Finally, Section 6 concludes the study.

2. Precious Metals

Precious metal prices respond to both short-term and long-term factors. Gold is a primary form of reserve asset held by central banks around the world. It is influential on other precious metal prices. Sari, Hammoudeh and Soytas (2010) states: "Among the major precious metal class, an increase in gold price seems to lead to parallel movements in the prices of other precious metals which are also considered investment assets as well as industrial commodities." The statement suggests that a model that adequately explains gold prices could also contribute to models used in predicting the prices of other precious metals. Hence, many economists consider gold as a leading indicator in the precious metal pack.

Silver is primarily obtained as a by-product of gold mining since two-thirds of the total world silver supply comes from this source. As a result, the price of silver is strongly related to that of gold (Hiller, Draper and Faff, 2006). Platinum is also jointly extracted with other metals such as palladium, rhodium, nickel and chrome. Demand for platinum comes mainly from industry for the conduction of catalytic converters for automobiles. Platinum is not held by central banks in the form of reserves, and therefore, the market for this metal is not directly sensitive to central banks' actions (Hiller et al., 2006).

There has been a growing interest in precious metal markets by agents that incorporate metals in production processes, such as many metallurgic companies, and the jewellery industry, where metals such as gold, platinum and silver are clearly dominant. These characteristics imply that there has been a strong demand in these markets. Most research done focused on the analysis of the gold market. The main interest has been the role of this precious metal as a hedge against inflation. Little has been done with regards to other precious metals (e.g., silver, platinum, palladium, etc.). Hiller et al. (2006) concluded that financial portfolios which contain precious metals perform significantly better than standard equity portfolios. They also found that precious metals exhibit some hedging capability during periods of abnormal volatility. Sari et al. (2010) examined the co-movements and information transmission among the spot prices of four precious metals (gold, silver, platinum and palladium), oil price and the US Dollar/Euro exchange rate. They found evidence of a weak long-run equilibrium relationship and strong feedbacks in the short run.

Hammoudeh, Malik and McAleer (2011) examined volatility and correlation dynamics in price returns of gold, silver, platinum and palladium, and explored the corresponding risk management implications for market risk and hedging. They used RiskMetrics and GARCH models in their analysis and concluded that the GARCH-t model should be used to calculate VaR in precious metals. Chaithep, Sriboonchitta, Chaiboonsri and Pastpipatkul (2012) used GEVD for risk evaluation of gold price returns and the tail distribution of extreme events in gold price returns. According to Chkili, Hammoudeh and Nguyen (2014), the FIAPARCH model is best suited for estimating VaR forecasts of commodity prices. They used daily spot and three-month futures of WTI, Henry Hub natural gas, gold and silver values from January 1997 to March 2011. Chen and Giles (2014) analysed the risk of investment in gold, silver and platinum by applying EVT to historical data for the changes in their prices. VaR and ES (or Conditional VaR - CVaR) estimates were obtained by fitting the GPD, using the POT method, to the extreme daily price changes. Chinhamu, Huang and Chikobvu (2015) evaluated risk in daily gold returns using generalised hyperbolic distributions (GHD) and the stable distribution. They found that the performances of GHDs and the stable distribution, in terms of VaR estimation, are comparable for gold price returns.

3. Distributions

3.1. The generalised lambda distribution (GLD)

GLD is a versatile distribution that can accommodate for a wide range of shapes, including fat-tailedness and asymmetry. GLD (Freimer, Kollia, Mudholkar and Lin, 1988) is defined by the following inverse cumulative distribution function:

$$F^{-1}(\mu,\lambda_1,\lambda_2,\lambda_3,\lambda_4)=\lambda_1+\frac{1}{\lambda_2}\left(\frac{\mu^{\lambda_3}-1}{\lambda_3}-\frac{(1-\mu)^{\lambda_4}}{\lambda_4}\right), \tag{1}$$

where $0\leq\mu\leq 1$; λ_1 is the location parameter, λ_2 is the scale parameter, λ_3 and λ_4 are related to skewness and kurtosis, respectively. For $\lambda_2>0$, this Freimer-Kollia-Mudholkar-Lin (FKML) parameterisation of GLD is well defined for all parameter values. Also, for a finite k-th moment of the GLD to exist, we must have $\min(\lambda_3,\lambda_4)>-\frac{1}{k}$.

Freimer et al. (1988) classified the shapes returned by (1) as follows:

(i) Class-I family $(\lambda_3<1,\lambda_4<1)$ represents unimodal densities with continuous tails.

(ii) Class-II family $(\lambda_3>1,\lambda_4<1)$ represents monotone probability density functions (pdfs) similar to the exponential distribution.

(iii) Class-III family $(1<\lambda_3<2,1<\lambda_4<2)$ represents U-shaped densities with truncated tails.

(iv) Class-IV family $(\lambda_3>2,1<\lambda_4<2)$ represents S-shaped densities.

(v) Class-V family $(\lambda_3>2,\lambda_4>2)$ represents unimodal densities with truncated tails.

3.1.1. Maximum likelihood estimation for the GLD

Several fitting methods for estimating parameters of the GLD have been proposed due to its flexibility in depicting various distributional shapes (Corlu and Meterelliyoz, 2016). In this paper, we utilise the maximum likelihood estimation procedure of Su (2007). The procedure uses the method of moments for the FKML parameterisation of GLD to find the initial values. These initial values are then used to maximise the numerical log-likelihood to find the parameters of the appropriate GLD. The algorithm is described below:

- Specify a range of initial values for λ_3,λ_4 and the number of initial values to be selected (λ_3,λ_4 are set by default to a range from -0.25 to 1.5 for the FKML GLD method of moments (Su, 2007)).
- Evaluate λ_1,λ_2 for each of the initial values of λ_3,λ_4. Remove all the set initial values that do not:
  - (a) result in a legal parameterisation of GLD.
  - (b) span the entire region of the data set.
- Calculate the quantiles μ_i from the initial values. This can be done by solving expression (1) numerically.

- Once μ_i is obtained, substitute μ_i into the numerical log-likelihood (2). The numerical log-likelihood can be obtained by applying the chain rule to differentiate expression (1) to obtain $f(x_i)$ and apply the logarithm on the joint distribution of $f(x_i)$, assuming independence of $f(x_i)$;

$$ML = \sum_{i=1}^{n} \log \left[\frac{\lambda_2}{\lambda_3 \mu_i^{\lambda_3 - 1} + \lambda_4 (1 - \mu_i)^{\lambda_4 - 1}} \right]. \tag{2}$$

- The optimal result can be obtained via the Nelder-Mead simplex algorithm, or other suitable numerical optimisation algorithms. It is always desirable to find another set of initial values in the optimisation process to check if the results obtained is a reasonable solution. The final fitting result can be examined using plots; histogram with the fitted line and quantile plot as well as testing the goodness of fit using resample Kolmogorov-Smirnov tests.

3.2. Extreme value theory (EVT)

EVT relates to the asymptotic behaviour of extreme observations of a random variable. It provides the fundamentals for the statistical modelling of rare events and is used to assess tail risk measures (Ren and Giles, 2010). The two fundamental methods for identifying extremes in EVT are the POT and the block maxima approaches (Coles, 2001; Gilli and Këllezi, 2006). The first method identifies realised large values over a predefined threshold and describes the behaviour of these exceedances by GPD. This is based on the fact that, given a large enough threshold, the distribution of these exceedances asymptotically tends toward the GPD. The latter method concentrates on the distribution of block maxima, which can be modelled by GEVD (since GEVD asymptotically describes the limiting behaviour of block maxima) (Coles, 2001).

3.2.1. The generalised Pareto distribution (GPD)

The two-parameter GPD (with scale parameter β and shape parameter ξ) has the following representation (Tsay, 2013):

$$G_{\xi.\beta}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - \exp(-\frac{y}{\beta}) & \text{if } \xi = 0 \end{cases} , \tag{3}$$

where $y > 0$ when $\xi \geq 0$, $0 \leq y \leq -\frac{\beta}{\xi}$ when $\xi < 0$, and $\beta > 0$.

Excess distribution

For a random variable Y, the excess distribution function F_u above a certain threshold u is defined as

$$F_u(y) = P(Y - u \leq y | Y > u),$$

where y represents the size of exceedances over u. Furthermore, if we denote F as the distribution function of Y, then we can write

$$F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)} . \tag{4}$$

A fundamental theorem in EVT, by Balkema and De Haan (1974) and Pickands (1975), identifies the asymptotic behaviour of these exceedances with GPD. Hence, the excess distribution function F_u can be well approximated by GPD for large enough u.

Peaks-over-threshold (POT)

To fit GPD to a data set, we adopt the POT method that focuses on the distribution of exceedances above some high threshold u. For $y-u \geq 0$, we can rewrite the excess distribution function (4) as

$$F_u(y-u) = \frac{F(y)-F(u)}{1-F(u)}$$

and, hence, deduce the following reverse expression

$$F(y) = (1-F(u))F_u(y-u)+F(u) \ ,$$

which allows us to apply the POT method. There are two steps in applying the POT method. Firstly, we need to choose an appropriate threshold. Secondly, we need to fit the GPD function to the data set. Given the choice of a sufficiently high threshold, we may estimate $F(u)$ by $\left(1-\frac{N_u}{n}\right)$, where n is the total sample size and N_u is the amount of observations above the chosen threshold. And, $F_u(y-u)$ can be estimated by a GPD using maximum likelihood estimation (MLE) (Embrechts et al., 1997). We then obtain the following tail estimator (Ren and Giles, 2010)

$$\hat{F}(y) = 1-\frac{N_u}{n}\left(1+\frac{\hat{\xi}}{\hat{\beta}}(y-u)\right)^{-1/\hat{\xi}} .$$

Threshold selection

In this paper we utilise the empirical mean excess plot for threshold selections. For a random variable Y, the mean excess function is defined as

$$e(u) = E(Y-u|Y>u) \ ,$$

i.e., the mean of exceedances over a threshold u. If the underlying distribution of $Y-u|Y>u$ follows a GPD, then the corresponding mean excess function is

$$e(u) = \frac{\beta+\xi u}{1-\xi} \ , \tag{5}$$

provided that $\beta+\xi u > 0$ and $\xi < 1$. From (5), we can clearly see that the mean excess function must be linear in u. More precisely, $Y > u$ follows a GPD if, and only if, the mean excess function is linear in u (Coles, 2001). This gives us a way of selecting an appropriate threshold. Given the data set, we define the empirical mean excess function as

$$e_n(u) = \frac{\sum_i^n (Y_i-u)I_{\{Y_i>u\}}}{\sum_i^n I_{\{Y_i>u\}}} \ ,$$

where n is the sample size. The empirical excess plot is a graphical representation of the locus of $(u, e_n(u))$ and we can examine this plot to choose the threshold u such that $e_n(u)$ is approximately linear for $Y > u$.

Parameter estimation

There are various techniques for estimating the parameters of the GPD, such as MLE, method of moments and the method of probability-weighted moments. We adopt the MLE method in this paper because the maximum likelihood estimator is asymptotically normal and allows simple approximations for standard errors and confidence intervals (Azzalini, 1996). Given that we have a sufficiently high threshold u and, assuming there are m observations with $y_i - u \geq 0$, the subsample $\{y_1 - u, \ldots, y_m - u\}$ has an underlying distribution of GPD, where $y_i - u \geq 0$ for $\xi \geq 0$, $0 \leq y_i - u \leq -\frac{\beta}{\xi}$ for $\xi < 0$, then the logarithm of the probability density function of $y_i - u$ can be derived from (3) as

$$\ln f(y_i - u) = \begin{cases} -\ln(\beta) - \frac{1+\xi}{\xi} \ln\left(1 + \xi\left(\frac{y_i - u}{\beta}\right)\right) & \text{if } \xi \neq 0 \\ -\ln(\beta) - \frac{1}{\beta}(y_i - u) & \text{if } \xi = 0 \end{cases} .$$

Hence, the log-likelihood function $L(\xi, \beta | y_i - u)$ for the GPD is the logarithm of the joint density of the m observations, i.e.

$$L(\xi, \beta | y_i - u) = \begin{cases} -m\ln(\beta) - \dfrac{1+\xi}{\xi} \sum_{i=1}^{m} \ln\left(1 + \xi\left(\dfrac{y_i - u}{\beta}\right)\right) & \text{if } \xi \neq 0 \\ -m\ln(\beta) - \frac{1}{\beta}\sum_{i=1}^{m}(y_i - u) & \text{if } \xi = 0 \end{cases} .$$

Therefore, we obtain the estimates for ξ and β by maximising the log-likelihood function of the subsample under a suitable threshold u.

Model validation

We can use quantile plots to assess the adequacy of a fitted generalised Pareto model (Coles, 2001). With a chosen threshold u, the ordered excesses $y_{(1)} \leq \cdots \leq y_{(m)}$ and an estimated model $\hat{F}$ with $\hat{\xi} \neq 0$, the quantile (Q-Q) plot consists of the pairs

$$\left\{ \left(\hat{F}^{-1}\left(\frac{i}{m+1}\right), y_{(i)} \right); i = 1 \ldots, m \right\},$$

where

$$\hat{F}^{-1}(y) = u + \frac{\hat{\beta}}{\hat{\xi}}\left[y^{-\hat{\xi}} - 1\right].$$

If GPD is a reasonable fit for the exceedances above u, then the Q-Q plot should depict points that are approximately linear. Furthermore, we may confirm the goodness-of-fit of GPD by utilising the excess distribution plot and plot of the tail of the underlying distribution (McNeil, Frey and Embrechts, 2005). For a good fit, the exceedances should lie close to the theoretical curves. Lastly, a scatter plot of residuals should not depict any visible pattern to indicate independence of the exceedances.

3.2.2. The generalised extreme value distribution (GEVD)

The GEVD is a family of continuous probability distributions. It includes the Gumbel, Fréchet and Weibull classes of distributions into a single family to allow a continuous range of possible shapes. Based on the extreme value theorem, GEVD is the limiting distribution of properly normalised maxima of a sequence of independent and identically distributed (i.i.d.) random variables (Coles, 2001). Thus, GEVD is used to model the maxima of a long (finite) sequence of random variables. The unified GEVD for modelling maxima is given by

$$G_{\xi,\mu,\sigma}(y) = \exp\left\{-\left(1+\xi\left(\frac{y-\mu}{\sigma}\right)\right)^{-1/\xi}\right\}, \tag{6}$$

with $\xi \neq 0, \sigma > 0$ and $1+\xi(\frac{y-\mu}{\sigma}) > 0$. The probability density function, obtained as the derivative of the distribution function (6), is given by

$$g_{\xi,\mu,\sigma}(y) = \frac{1}{\sigma}\left(1+\xi\left(\frac{y-\mu}{\sigma}\right)\right)^{-1-1/\xi} \exp\left\{-\left(1+\xi\left(\frac{y-\mu}{\sigma}\right)\right)^{-1/\xi}\right\},$$

for $\xi \neq 0$ and where μ and σ are the location and scale parameters, respectively. The shape parameter ξ is also known as the extreme value index (EVI). ξ^{-1} is the rate of tail decay of the GEVD distribution. If $\xi > 0$, G belongs to the heavy-tailed Fréchet class of distributions such as Pareto, Cauchy, Student-t and mixture distributions. If $\xi < 0$, G belongs to the short-tailed Weibull class of distributions such as uniform and beta distributions. If $\xi = 0$, then G belongs to the light-tailed Gumbell class of distributions and includes distributions such as normal, exponential, gamma and lognormal distributions (Bali, 2003).

Block maxima

The block maxima method is an EVT approach for identifying extremes in a data set and for describing their asymptotic behaviour. It consists of fitting GEVD to a particular set of maxima chosen in a given sample of data. This method may be summarised as follows.

Given a sample $Y_1, Y_2, \ldots, Y_n$ of i.i.d. data points drawn from an unknown distribution F:

(a) We divide the sample into m non-overlapping subsamples of b observations each (b-blocks), for given integers m and b $(0 < m, b < n)$, and denote $M_{(b,j)}$ as the maximum of the j^{th} subsample;

(b) Assuming that F belongs to the maximum domain of attraction of $G_{\xi,\mu,\sigma}$ for some $\xi, \mu, \sigma \in \mathbb{R}$, with $\sigma > 0$, we fit GEVD to the sequence of block maxima $M_{(b,1)}, \ldots, M_{(b,m)}$ to determine the parameter estimates $\hat{\xi}$, $\hat{\mu}$ and $\hat{\sigma}$ (by, for example, using MLE).

Analogous results and methods can be employed to study the asymptotic distribution of minima, taking into account the relation $\min\{Y_1, \ldots, Y_n\} = \max\{-Y_1, \ldots, -Y_n\}$. In the context of this paper, we use block sizes of 5, 10 and 21, corresponding to weekly, fortnightly and monthly maxima.

Maximum likelihood estimation for GEVD

Under the assumption that $Y_1,\ldots,Y_m$ are independent variables following the GEVD, the log-likelihood for the GEVD parameters, when $\xi \neq 0$, is

$$\ell(\mu,\sigma,\xi) = -m\ln\sigma - (1+\frac{1}{\xi})\sum_{i=1}^{m}\ln\left[1+\xi\left(\frac{y_i-\mu}{\sigma}\right)\right] - \sum_{i=1}^{m}\left[1+\xi\left(\frac{y_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}, \tag{7}$$

provided that $1+\xi(\frac{y_i-\mu}{\sigma}) > 0$ for $i = 1,\ldots,m$. For parameter combinations in which the above condition is violated, i.e., a configuration for which at least one of the observed data point falls beyond an end-point of the distribution, the likelihood is zero and the log-likelihood equals $-\infty$. The case $\xi = 0$ requires separate treatment using the Gumbel limit of GEVD. This leads to the log-likelihood

$$\ell(\mu,\sigma) = -m\ln\sigma - \sum_{i=1}^{m}\left(\frac{y_i-\mu}{\sigma}\right) - \sum_{i=1}^{m}\exp\left\{-\left(\frac{x_i-\mu}{\sigma}\right)\right\}. \tag{8}$$

Maximisation of (7) and (8), with respect to the parameter vector, leads to the maximum likelihood estimates for the entire GEVD family (Coles, 2001). MLE offers the advantage of simultaneous estimation of the three parameters.

Model checking for GEVD

After estimating the GEVD parameters, one can check the goodness-of-fit of the model by utilising residual plots, where the residuals are defined as:

$$res = \begin{cases} \left(1+\frac{\xi}{\sigma}(y-\mu)\right)^{-1/\xi} & \text{if } \xi = 0 \\ \exp\left[-\exp\left(-\frac{y-\mu}{\sigma}\right)\right] & \text{if } \xi \neq 0 \end{cases}.$$

The data points are converted to unit exponentially distributed residuals under the null hypothesis that GEVD provides a good depiction of the data set.

4. Risk Measures

The amount of asset risk capital, reserved by financial institutes as per the Basel agreement, is directly associated to the portfolio risk level and two of the most common benchmark measures for evaluating such risk are VaR and ES. VaR is intended to assess the maximum possible loss of a portfolio over a given time period, given a certain risk level, and its calculations focus on the tails of a distribution. Whereas ES evaluates the expected value of losses (or gains) that exceed a corresponding VaR level. Hence, the accuracies of VaR and ES estimation depend on how well a selected model portrays the extreme data observations (McNeil et al., 2005).

Value-at-Risk

VaR has become a benchmark for evaluating market risks. There are two main approaches to calculating VaR for financial data series; the parametric method and the non-parametric method (Brooks and Persand, 2000). In this paper we estimate VaR using the proposed distributions (parametric approach for GLD and semi-parametric approaches for GPD and GEVD). For a random variable Y (usually for modelling the return distribution of some risky financial portfolio) with distribution function F over a specified time period, the VaR (for a given probability p) can be defined as the p^{th} quantile of F, i.e.

$$VaR_p = F^{-1}(1-p) \, ,$$

where F^{-1} is the quantile function.

GLD is represented with a quantile function (Corlu and Corlu, 2015), hence

$$\widehat{VaR}_p = F^{-1}(p, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4) = \hat{\lambda}_1 + \frac{1}{\hat{\lambda}_2}\left(\frac{p^{\hat{\lambda}_3}-1}{\hat{\lambda}_3} - \frac{(1-p)^{\hat{\lambda}_4}-1}{\hat{\lambda}_4}\right) .$$

We may also use GEVD and GPD to model and approximate this risk measure. For a small upper tail probability p, the GEVD approximation to VaR can be written as

$$\widehat{VaR}_p = \begin{cases} \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}}\left\{1-[-n\ln(1-p)]^{-\hat{\xi}}\right\} & \text{if } \hat{\xi} \neq 0 \\ \hat{\mu} - \hat{\sigma}\ln[-n\ln(1-p)] & \text{if } \hat{\xi} = 0 \end{cases} ,$$

where n is the length of the subperiod and $\hat{\mu}$, $\hat{\sigma}$ and $\hat{\xi}$ are the maximum likelihood estimates of the GEVD parameters, and the GPD approximation to VaR is given by

$$\widehat{VaR}_p = \begin{cases} u + \frac{\hat{\beta}}{\hat{\xi}}\left\{\left(\frac{n}{N_u}p\right)^{-\hat{\xi}} - 1\right\} & \text{if } \hat{\xi} \neq 0 \\ u - \hat{\beta}\ln\left(\frac{n}{N_u}(1-p)\right) & \text{if } \hat{\xi} = 0 \end{cases} , \tag{9}$$

where $\hat{\beta}$ and $\hat{\xi}$ are the estimates of the GPD parameters and N_u is the number of exceedances above the threshold u in a given sample (Tsay, 2010).

Expected Shortfall

Although VaR is often considered as an adequate risk measure, it does not capture all aspects of market risks, such as subadditivity. Hence, Artzner, Delbaen, Eber and Heath (1999) proposed ES as a better measure of risk, which is subadditive and also informs one about the likely magnitude of exceedances. ES gives the expected size of return that exceeds VaR, i.e., for a probability level p,

$$ES_p = E(X|X > VaR_p) \, .$$

And, equivalently,

$$ES_p = VaR_p + E(X - VaR_p|X > VaR_p) \, ,$$

where the second term above represent the mean of the excess distribution $F_{VaR_p}(x)$ (treating VaR_p as the threshold). Proceeding as before, if the threshold VaR_p is sufficiently large, then $F_{VaR_p}(x)$ is a GPD, i.e.

$$F_{VaR_p}(x) = G_{\xi,\beta+\xi(VaR_p-u)}(x)\ .$$

Thus, the mean of the excess distribution $F_{VaR_p}(x)$ can be calculated as

$$\frac{\beta+\xi(VaR_p-u)}{1-\xi}\ ,$$

where $\xi < 1$, and substituting into equation (9) yields

$$\widehat{ES}_p = \frac{\widehat{VaR}_p}{1-\hat{\xi}} + \frac{\hat{\beta}-\hat{\xi}u}{1-\hat{\xi}}.$$

ES for GLD is given by (Corlu and Corlu, 2015)

$$\begin{aligned}\widehat{ES}(p) &= \frac{1}{p}\int_0^p F^{-1}(p,\lambda_1,\lambda_2,\lambda_3,\lambda_4)\,dy \\ &= \lambda_1 + \frac{1}{p\lambda_2\lambda_3}\left(\frac{p^{\lambda_3+1}}{\lambda_3+1}-p\right) - \frac{1}{p\lambda_2\lambda_4}\left(\frac{1-(1-p)^{\lambda_4+1}}{\lambda_4+1}-p\right).\end{aligned}$$

Backtesting

To examine the adequacy and effectiveness of VaR and ES estimates, we utilise various backtesting procedures. In particular, VaR backtesting is performed using the Kupiec likelihood ratio unconditional coverage test (Kupiec, 1995) and Christoffersen conditional coverage test (Christoffersen, 1998). For ES, we follow the backtesting procedure in McNeil and Frey (2000), implemented for both with and without bootstrapping. The Kupiec test exploits the fact that an adequate model ought to have its proportion of violations of VaR estimates close to the corresponding tail probability level. The method consists of calculating the number of times, x^p, the observed returns fall below (for long positions) or above (for short positions) the VaR estimate at level p, i.e., $r_t < VaR_p$ or $r_t > VaR_p$, and compare the corresponding failure rates to p. The null hypothesis is that the expected proportion of violations is equal to p. Under this null hypothesis, the Kupiec statistic, which is given by

$$LR_{UC} = 2\ln\left(\left(\frac{x^p}{N}\right)^{x^p}\left(1-\frac{x^p}{N}\right)^{N-x^p}\right) - 2\ln(p^{x^p}(1-p)^{N-x^p}),$$

is asymptotically distributed according to a chi-square distribution with one degree of freedom. The Christoffersen test extends the Kupiec test and accounts for serial independence of violations (i.e., clustering of extremes). The Christoffersen test statistic is given by

$$LR_{CC} = LR_{UC} + 2\ln\frac{\left[(1-\pi_0)^{\phi_{00}}\pi_0^{\phi_{01}}(1-\pi_1)^{\phi_{10}}\pi_1^{\phi_{11}}\right]}{\left[(1-\pi)^{(\phi_{00}+\phi_{10})}\pi^{(\phi_{01}+\phi_{11})}\right]}\ ,$$

where ϕ_{ij} is defined as the number of returns in state i while they have been in state j previously (state 1 indicates that the VaR estimate is violated and state 0 indicates that it is not) and π_i is defined as the probability of having an exception that is conditional on state i the previous day. This statistic is asymptotically chi-square distributed with two degrees of freedom.

The null hypothesis of the ES backtest is that the excess conditional shortfalls (excess of the actual data series when VaR is violated), are i.i.d. and has zero mean. The test is a one sided t-test against the alternative that the excess shortfall has a mean greater than zero, and thus the conditional shortfall is systematically underestimated. The test statistic is given by

$$T = \frac{\bar{r} - \mu_0}{\bar{\sigma}/\sqrt{m}} ,$$

where $\bar{r}$ and $\bar{\sigma}$ are the mean and standard deviation of "exceedance residuals" $\{r_1, r_2, \ldots, r_m\}$. The bootstrap techniques can also be utilised to alleviate any bias with respect to assumptions about the underlying distribution of the excess shortfall. For the bootstrap test, we sample $\{\tilde{r}_1^*, \tilde{r}_2^*, \ldots, \tilde{r}_m^*\}$ without replacement from the shifted residuals $\tilde{r}_i = r_i - \bar{r} + \mu_0$ and compute the test statistic

$$T_j^* = \frac{\bar{\tilde{r}}^* - \mu_0}{\bar{\tilde{\sigma}}/\sqrt{m}}$$

for each bootstrap sample j (McNeil and Frey, 2000).

5. Empirical Results

5.1. Data

The data sets used in this study are the daily gold, platinum and silver prices obtained from McGregor BFA and were recorded over the period from 2 April 1990 to 18 September 2014. The return series for each index are calculated as the first backward-differences of the natural logarithm of the index values. For day t, the daily log return r_t is defined as

$$r_t = \ln(P_t) - \ln(P_{t-1}) ,$$

where P_t is the price at day t. For financial risk assessments, we are primarily interested in the losses, i.e. negative returns. Results and methods for GEVD and GPD can be employed to study the asymptotic distribution of extreme losses, taking into account the relation $\min\{X_1, \ldots, X_n\} = \max\{-X_1, \ldots, -X_n\}$.

Figures 1, 2 and 3 are time series plots of daily precious metal prices and returns for over 25 years. The plots strongly indicate the presence of heteroscedasticity and volatility clustering in all return series. Isolated extreme returns caused by shocks to financial markets are visible, such as the 2009 financial crisis.

A descriptive summary of the different precious metal returns is provided in Table 1. All returns have a positive mean indicating that the overall returns were slightly increasing over the period under investigation. The excess kurtosis value indicates the leptokurtic behaviour of these return series. This means that the empirical distribution of the daily precious metal returns are much fatter than the normal distribution. The Anderson-Darling test for normality gives a p-value less than 0.0001 for all three precious metals returns rejecting the normality assumption at all levels of significance.

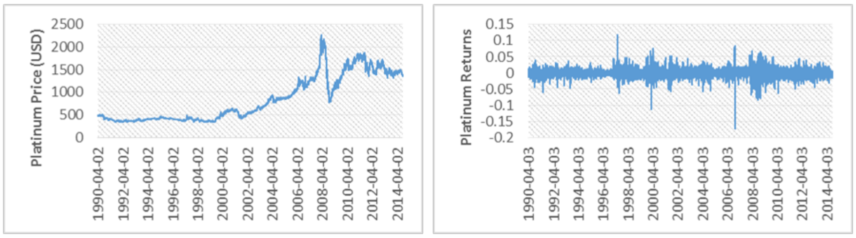


Figure 1: Time series plot of platinum prices (left) and one-day returns (right).

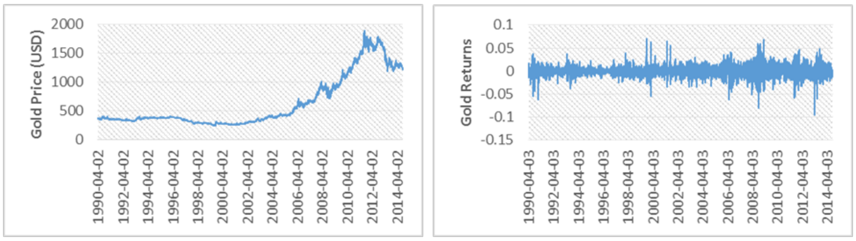


Figure 2: Time series plot of gold prices (left) and one-day returns (right).

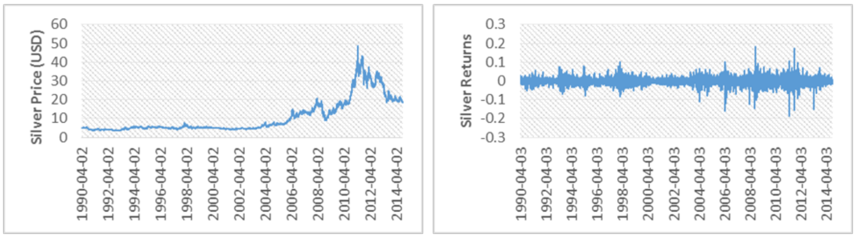


Figure 3: Time series plot of silver prices (left) and one-day returns (right).

Table 1: Descriptive statistics of precious metal price returns.

	Min	Max	Mean	Std. dev.	Skewness	Excess kurtosis
Platinum	-0.17280	0.11780	0.0002246	0.01378826	-0.5253288	10.19592
Gold	-0.07972	0.07006	0.0002786	0.01012477	-0.105173	6.202978
Silver	-0.18690	0.18280	0.0003428	0.01958386	-0.3642794	9.671625

5.2. Results and discussions on fitted distributions

We fitted the GLD, GPD and GEVD to each of the three precious metal data sets by using maximum likelihood estimation. The results are presented and discussed in the subsequent sections.

Platinum returns

The block maxima of the negative returns have been fitted to GEVD with different block sizes (5, 10, and 21). Table 2 shows the maximum likelihood estimates of the parameters with increasing block sizes and their corresponding standard errors (SE).

Table 2: Parameter estimates using GEVD for platinum returns.

	No. of Maxima	ξ	SE	σ	SE	μ	SE
GEVD5	1127	0.138587	0.018131	0.007414	0.000157	0.008521	0.000240
GEVD10	564	0.250289	0.034781	0.007296	0.000241	0.012565	0.000343
GEVD21	269	0.225674	0.044754	0.008736	0.000423	0.017721	0.000586

The shape parameter is positive for all cases. Hence, the extreme negative returns follow a Fréchet family of GEVD for all block sizes considered. The Fréchet type of GEVD confirms that the original series has a negative fat tail.

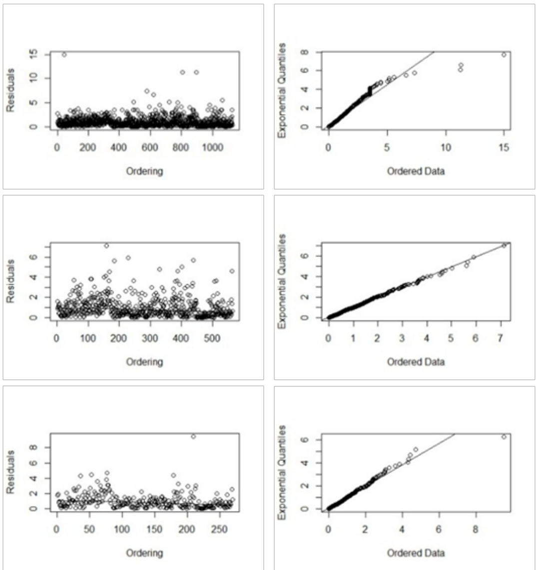


Figure 4: Residual and Q-Q plots for negative platinum returns with block size 5, 10 and 21.

In Figure 4 we observe that the residuals are approximately exponentially distributed with most points lying on straight lines (recall that the residuals are converted to exponential residuals and the straight line indicates perfectly exponentially distributed behaviour), indicating that GEVD fits the data set well.

The mean excess plots determine a suitable threshold which is necessary for fitting the GPD model. The choice of a threshold should be depicted by linear increases in the mean excess plot. Figure 5 presents the mean excess function of negative platinum returns. By observing the mean excess function in Figure 5, a threshold of between 0.1% and 2.5% seems to be a reasonable choice. We selected thresholds at 70th, 80th and 90th percentiles. These provided reasonable choices as they yield enough data points for analyses and they fall within the above range. The GPD parameter estimates for the chosen thresholds are given in Table 3. To assess the suitability of the GPD model for our data set, one can use the plot of excess distribution and Q-Q plot of the residuals. Figure 6 confirms a good fit of the GPD at all three chosen thresholds.

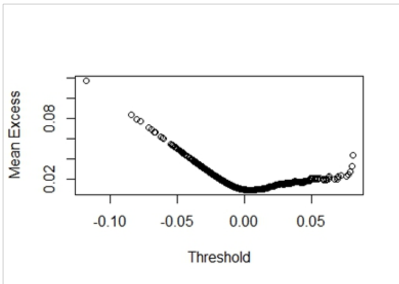


Figure 5: Mean excess function of negative platinum returns.

Table 3: Parameter estimates using GEVD for platinum returns.

	Threshold	No. of Exceedances	ξ	SE	β	SE
GPD70	0.004662	1690	0.127558	0.024935	0.008131	0.000271
GPD80	0.008302	1127	0.185612	0.033915	0.007725	0.000331
GPD90	0.014279	564	0.266063	0.054846	0.007728	0.000506

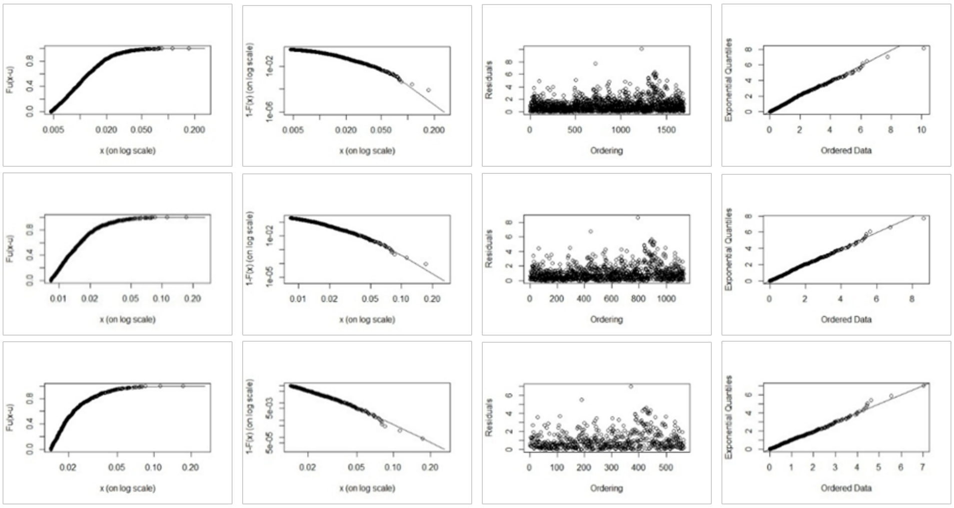


Figure 6: Diagnostic plots for negative platinum returns fitted with GPD and a threshold of 0.004662, 0.008302 and 0.014279 respectively.

Based on results in Tables 4, 9 and 14, all three precious metals have λ_3 and λ_4 less than 1, i.e., their densities are unimodal with continuous tails, i.e. class-I family of GLD. The histogram and Q-Q plots in Figure 7 indicate a good fit of GLD for platinum returns.

Table 4: GLD parameter estimates of platinum returns.

	λ_1	λ_2	λ_3	λ_4
GLD	-0.000035	189.0246	-0.168427	-0.193859

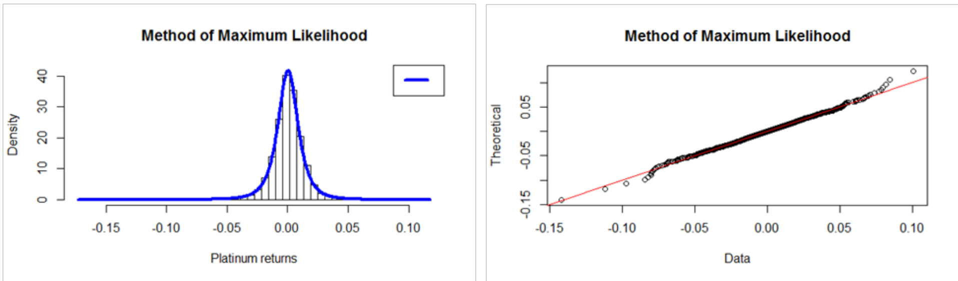


Figure 7: Histogram and Q-Q plots for GLD on platinum returns.

Table 5: VaR and ES estimates for platinum returns using heavy-tailed distributions.

	VaR		**ES**	
Model	**1%**	**0.1%**	**1%**	**0.1%**
GEVD5	0.035997	0.066505	0.052546	0.085133
GEVD10	0.035221	0.075705	0.051078	0.097957
GEVD21	0.034001	0.071566	0.049751	0.095156
GPD70	0.039288	0.072871	0.056460	0.095156
GPD80	0.039262	0.077964	0.056460	0.101456
GPD90	0.038848	0.084169	0.055892	0.123423
GLD	0.039259	0.076799	0.056460	0.097957

Table 6: VaR and ES Backtesting for platinum returns using heavy-tailed distributions.

	No. of violations		**Kupiec *p*-value**		**Christoffersen *p*-value**		**t-test**		**Bootstrap t-test**	
Model	**1%**	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**
GEVD5	74	13	0.0240	0.0081	< 0.0001	< 0.0001	0.5000	0.5000	0.5547	0.5868
GEVD10	81	7	0.0019	0.5789	< 0.0001	0.8498	0.5000	0.5000	0.5303	0.5687
GEVD21	88	8	< 0.0001	0.3483	< 0.0001	0.6368	0.5000	0.5000	0.5216	0.5602
GPD70	59	8	0.7227	0.3483	< 0.0001	0.6368	0.5000	0.5000	0.5318	0.5610
GPD80	59	6	0.7227	0.8783	< 0.0001	0.9820	0.5000	0.5000	0.5409	0.5706
GPD90	61	3	0.5372	0.2227	< 0.0001	0.4747	0.5000	0.5000	0.5445	0.5368
GLD	59	7	0.7227	0.5789	< 0.0001	0.8498	0.5000	0.5000	0.5536	0.5764

Table 5 shows VaR and ES estimates for platinum returns using the three heavy-tailed distributions. GPD and GLD give the largest p-values at 1% VaR level, GPD gives the highest p-values at 0.1% VaR. It is interesting to note that all distributions were rejected at 1% VaR level using the Christoffersen test. However, at 0.1% all models are suitable except the GEVD (block size 5) and the best model being GPD. As for backtesting of ES, with and without bootstrapping, very suitable ES estimates are obtained for all three distributions both at 1% and 0.1%.

Gold returns

Parameter estimates of GEVD, GPD and GLD for gold returns are likewise obtained as for platinum returns. Results are presented in Tables 7 to 9 and Figures 8 to 11.

Table 7: Parameter estimates using GEVD.

	No. of Maxima	ξ	**SE**	σ	**SE**	μ	**SE**
GEVD5	1127	0.132949	0.019191	0.005735	0.000108	0.005883	0.000184
GEVD10	564	0.254529	0.039871	0.005818	0.000181	0.008803	0.000277
GEVD21	269	0.178242	0.052262	0.007196	0.000346	0.012848	0.000500

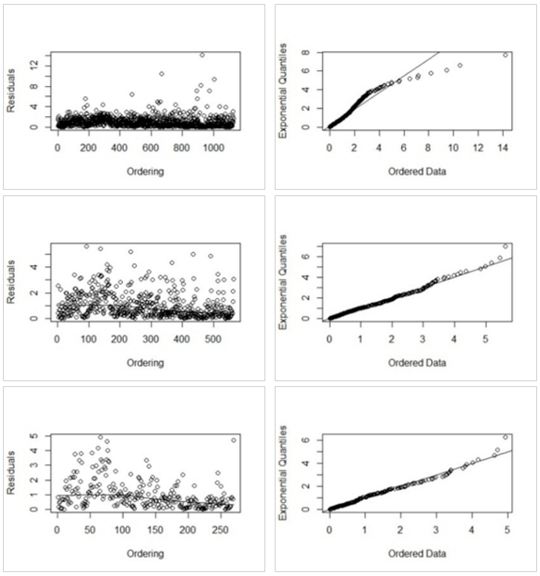


Figure 8: Residual and Q-Q plots for negative gold returns with block size 5, 10 and 21.

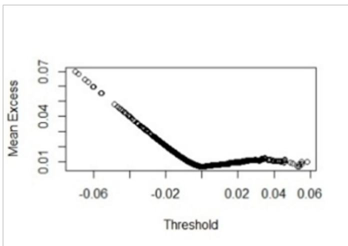


Figure 9: Mean excess function of negative gold returns.

Figure 9 presents the mean excess function of negative gold returns. By observing the mean excess function in Figure 9, a threshold of between 0.1% and 2% seems to be a reasonable choice for negative gold returns. Again, we make corresponding selection of threshold as for platinum returns, while conforming to the above range. The GPD parameter estimates for the chosen thresholds are given in Table 8.

Table 8: Parameter estimates using GEVD for gold returns.

	Threshold	**No. of Exceedances**	ξ	**SE**	β	**SE**
GPD70	0.002980	1690	0.121231	0.027124	0.006224	0.000210
GPD80	0.005544	1127	0.120456	0.033428	0.006557	0.000274
GPD90	0.010292	564	0.123112	0.048070	0.007088	0.000427

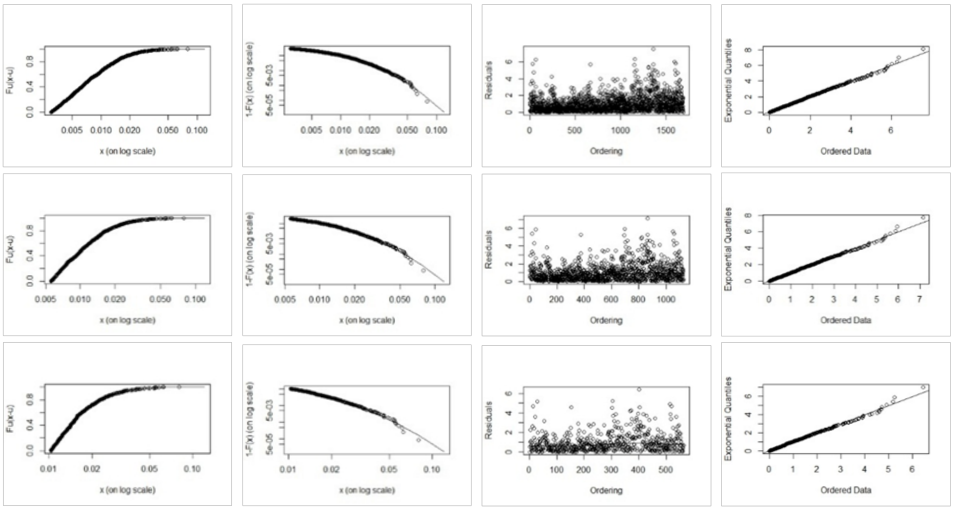


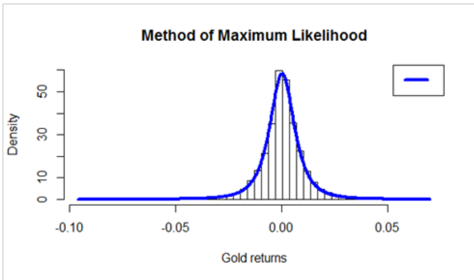
Figure 10: Diagnostic plots for negative gold returns fitted with GPD and a threshold of 0.002980, 0.005544 and 0.010292 respectively.

Table 9: GLD parameter estimates of gold returns.

	λ_1	λ_2	λ_3	λ_4
GLD	-0.000268	301.619722	-0.258695	-0.266889

The model checking plots again indicates good fit for the different models. The only exception are the deviations in GEVD5 and slight tail misfits for GLD.

Table 11 presents the backtesting results for VaR and ES estimates of gold returns using the three heavy-tailed distributions. The GPD, at all chosen thresholds, is the best model for most of the tests and at different VaR levels. In fact, it is the model that stands out in terms of both VaR and ES estimation. Meanwhile, GLD did not perform as well as it did for platinum returns.



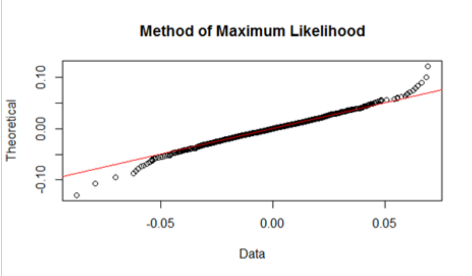


Figure 11: Histogram and Q-Q plots for GLD on gold returns.

Table 10: VaR and ES estimates for gold returns using heavy-tailed distributions.

Model	VaR 1%	VaR 0.1%	ES 1%	ES 0.1%
GEVD5	0.026944	0.049989	0.037844	0.058375
GEVD10	0.034882	0.073982	0.047246	0.079719
GEVD21	0.041277	0.076274	0.051549	0.079719
GPD70	0.029182	0.054152	0.039880	0.068294
GPD80	0.029200	0.054160	0.039895	0.068273
GPD90	0.029172	0.054227	0.039905	0.068479
GLD	0.029737	0.065808	0.040870	0.079719

Table 11: VaR and ES Backtesting for gold returns versus heavy-tailed distributions.

Model	No. of violations 1%	No. of violations 0.1%	Kupiec p-value 1%	Kupiec p-value 0.1%	Christoffersen p-value 1%	Christoffersen p-value 0.1%	t-test 1%	t-test 0.1%	Bootstrap t-test 1%	Bootstrap t-test 0.1%
GEVD5	70	10	0.0778	0.0974	0.0016	0.2486	0.5000	0.5000	0.5108	0.5613
GEVD10	30	1	0.0001	0.0159	0.0005	0.0547	0.5000	0.5000	0.5436	0.5761
GEVD21	20	1	< 0.0001	0.0159	< 0.0001	0.0547	0.5000	0.5000	0.5350	0.5342
GPD70	58	6	0.8239	0.8783	0.0118	0.9820	0.4993	0.9523	0.5234	0.8915
GPD80	58	6	0.8239	0.8783	0.0118	0.9820	0.5036	0.9518	0.5162	0.8813
GPD90	58	6	0.8239	0.8783	0.0118	0.9820	0.5067	0.9570	0.5228	0.8889
GLD	53	1	0.6524	0.0159	0.0448	0.0547	0.5000	0.5000	0.5272	0.5241

Silver returns

The corresponding results for silver returns are presented in Tables 12 to 16 and Figures 12 to 15. Figure 12 indicates some tail misfits for GEVD5, while the other two GEVDs seem to provide good general fits. However, this does not necessarily translate to good VaR and ES estimates, as we observed for gold returns.

By observing the mean excess function in Figure 13, a threshold of between 0.1% and 5% seems to be a reasonable choice for negative silver returns. The GPD parameter estimates for the chosen thresholds are given in Table 13. Figure 14 illustrates a good fit of the GPD at all three chosen thresholds.

Figure 15 indicates a good fit of GLD for silver returns.

Table 12: Parameter estimates using GEVD for silver returns.

	No. of Maxima	ξ	**SE**	σ	**SE**	μ	**SE**
GEVD5	1127	0.143616	0.017662	0.010289	0.000235	0.012165	0.000333
GEVD10	564	0.247685	0.034304	0.010613	0.000383	0.018135	0.000502
GEVD21	269	0.331125	0.058408	0.011621	0.000661	0.024157	0.000815

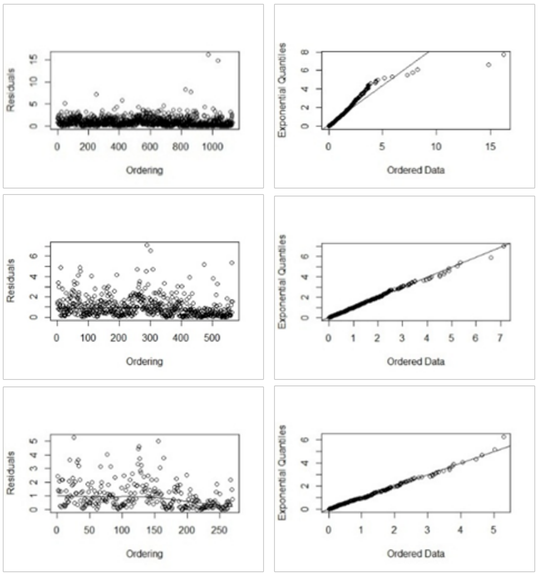


Figure 12: Residual and Q-Q plots for negative silver returns with block size 5, 10 and 21.

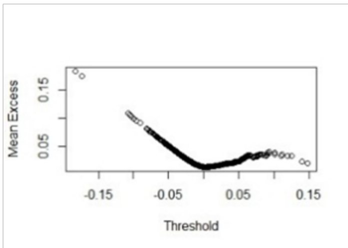


Figure 13: Mean excess function of negative silver returns.

Table 13: Parameter estimates using GPD for silver returns.

	Threshold	**No. of Exceedances**	ξ	**SE**	β	**SE**
GPD70	0.006320	1690	0.143453	0.026046	0.011354	0.000395
GPD80	0.011273	1127	0.181534	0.034096	0.011277	0.000497
GPD90	0.019885	564	0.238896	0.053594	0.011669	0.000772

Table 15 presents the VaR and ES estimates for silver returns using the three heavy-tailed distributions and the corresponding backtest results are provided in Table 16. For the Kupiec test, GLD

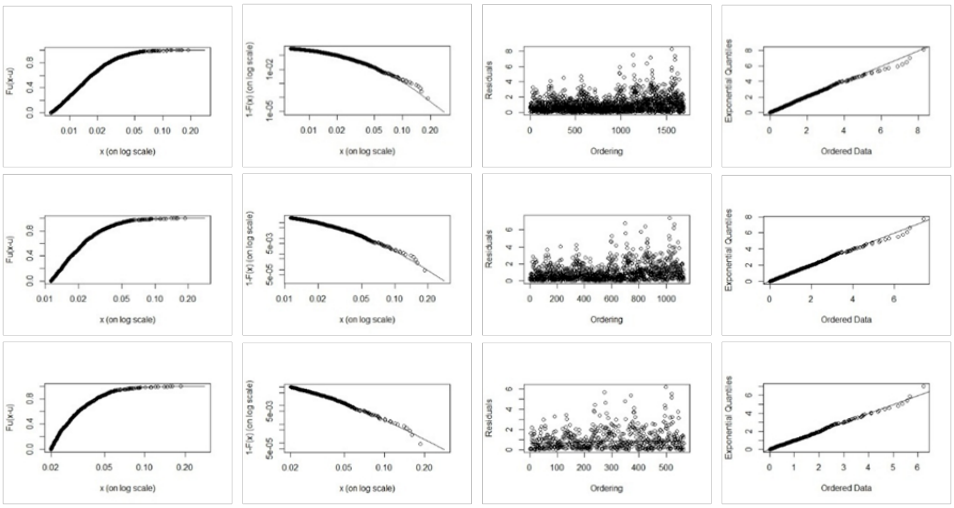


Figure 14: Diagnostic plots for negative silver returns fitted with GPD and a threshold of 0.006320, 0.011273 and 0.019885 respectively.

Table 14: GLD parameter estimates of silver returns.

	λ_1	λ_2	λ_3	λ_4
GLD	-0.000023	141.2906	-0.201810	-0.214

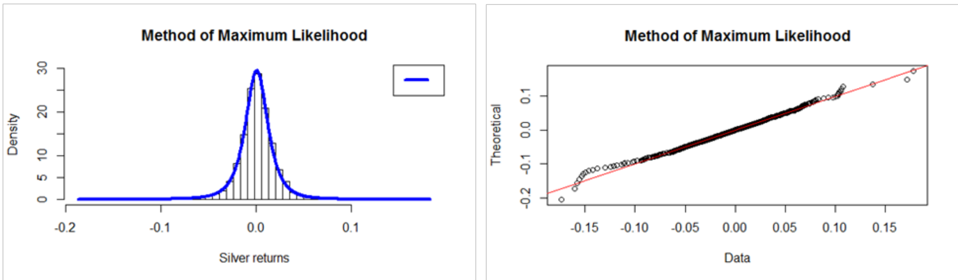


Figure 15: Histogram and Q-Q plots for GLD on silver returns.

provides the largest p-values at 1% VaR level and the GPD (at 90% threshold) gives the highest p-values at 0.1% VaR. It is interesting to note that all distributions were rejected at 1% VaR level using the Christoffersen test. However, at 0.1% all models are suitable except the GEVD (block size 5). As for backtesting of ES, results from test with and without bootstrapping indicate very suitable ES estimates for the three distributions both at 1% and 0.1%. Although GPD at 90% threshold is a clear winner at 0.1% level.

Table 15: VaR and ES estimates for silver returns using heavy-tailed distributions.

	VaR		ES	
Model	**1%**	**0.1%**	**1%**	**0.1%**
GEVD5	0.050603	0.093847	0.073359	0.132876
GEVD10	0.050985	0.109336	0.073661	0.139922
GEVD21	0.047806	0.115170	0.070833	0.148131
GPD70	0.056098	0.106560	0.077690	0.136603
GPD80	0.056168	0.111702	0.079905	0.147755
GPD90	0.055734	0.117848	0.082318	0.163928
GLD	0.055441	0.111932	0.081783	0.143613

Table 16: VaR and ES Backtesting for silver returns versus heavy-tailed distributions.

	No. of violations		Kupiec p-value		Christoffersen p-value		t-test		Bootstrap t-test	
	1%	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**	**1%**	**0.1%**
GEVD5	76	11	0.0124	0.0456	<0.0001	0.0071	0.5000	0.5000	0.5114	0.5169
GEVD10	75	9	0.0173	0.1920	<0.0001	0.0147	0.5000	0.5000	0.5340	0.4987
GEVD21	85	7	0.0003	0.5789	<0.0001	0.0173	.5000	0.5000	0.5152	0.5240
GPD70	50	9	0.3875	0.1920	0.0002	0.0147	0.0785	0.3480	0.1488	0.3826
GPD80	49	8	0.3154	0.3483	0.0002	0.0173	0.1527	0.6835	0.2301	0.6588
GPD90	52	7	0.5569	0.5789	0.0004	0.0173	0.4561	0.9677	0.4964	0.9158
GLD	54	8	0.7534	0.3483	<0.0001	0.0173	0.5000	0.5000	0.5254	0.5030

6. Conclusion

We have analysed daily log returns of precious metals (platinum, gold and silver) using three distributions, namely GEVD, GPD and GLD. The data sets are for the period 2 April 1994 to 18 September 2014. Our analyses show that GPD and GLD generally outperform the GEVD for VaR estimation of negative precious metal returns. For gold, GPD stands out as the most suitable model. For platinum, it is between GPD and GLD, especially at the 1% level. For silver, GLD is the most suitable at 1% level, whereas GPD is the best model at 0.1%. On the other hand, the difference in ES estimation between the three distributions for platinum is minor, while GPD stands out for ES estimation at the 0.1% level for both gold and silver.

The purpose of this study has been to compare the performance of the two EVT models to a flexible distribution, GLD, for describing daily log returns of precious metal prices. This paper confirms the results by Ren and Giles (2010) that EVT is a reliable method for predicting future potential extreme losses/gains for precious metals. It also shows that GLD competes favourably with EVT for predicting future potential extreme losses/gains for precious metal results, especially for silver and platinum returns. These results do not imply that GPD and GLD will always give good fits for every financial data set. As further research, we recommend out-of-sample backtests and comparisons with generalised POT models, such as DPOT and PORT, and the generalised logistic distribution for block maxima.

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