CO-OPERATIVE LEARNING AS A TOOL IN THE TEACHING AND LEARNING OF LINEAR PROGRAMMING: A CASE OF NATIONAL CURRICULUM VOCATIONAL (NC(V)) LEVEL 3 MULTILINGUAL STUDENTS

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ABSTRACT

This study explored how co-operative learning could be used in the teaching and learning of Linear Programming (LP) for NCV multilingual FET students. A qualitative case study was used in an FET college where students and their lecturers learn mathematics in a language that is not their first, main or home language. The data analysed showed that students had low language proficiency in the Language of Learning and Teaching (LoLT). This then calls for lecturers to apply different language practices, such as code-switching and co-operative learning, to enable students to understand the language in which tasks are presented. It is from understanding a task that students can employ different solution strategies to solve the problems presented to them. The data further revealed that code-switching and co-operative learning were practised as a teaching strategy to ensure an understanding of LP concepts in the lecturer's classroom. Home language students were able to translate mathematical statements to symbolic form. Code-switching and discussions in their home languages further enabled them to understand the procedures and strategies of solving LP problems.

Keywords: Linear programming, Language of Learning and Teaching (LoLT), code-switching, cooperative learning.

INTRODUCTION

This article investigated co-operative learning as a tool in the learning of Linear Programming (LP). The study further explored ways in which students' solution strategies linked with the language practices used in the teaching of LP. Various studies that focus on language practices have been done elsewhere (Barwell 2018; Moschkovich 2018; Phakeng 2018; Phakeng and Moschkovich 2013; Planas 2018). Since there are no studies that have focused on language practices and LP in FET colleges, this study explored these gaps and provides relevant recommendations. Our interest in this study was not to investigate poor performance in LP, but to investigate the strategies that students apply when solving LP, as well as the language practices employed during the teaching and learning of the word problems embedded in LP.

Thus, in this article, we argue that co-operative learning and language practices, such as code-switching, can enhance multilingual students' ability to solve LP tasks. To support this argument, we asked the following critical questions to guide our study: (1) How does co-operative learning enhance or hinder multilingual students' ability to solve LP tasks? (2) In what ways do their solution strategies link with language practices used in the teaching of LP? Furthermore, we discuss Situated Cognition as the theory that guided this study. We used this theory as a lens through which we interpreted and analysed the collected data. We also present the literature that was reviewed for this study, which indicates the importance of language in the teaching and learning of mathematics. It also highlights the relationship between mathematics and the Language of Learning and Teaching (LoLT), the importance of learners' fluency in the LoLT, and the challenges that second-language learners face when they learn mathematics in a language that is not their own. It emphasises the complexities associated with the language of mathematics, as well as the effect of teacher knowledge on students' conceptual understanding and achievement.

The purpose of LP in the curriculum policy

The Subject Guidelines for Mathematics Level 3 of the Department of Education (DoE 2011) advocates that mathematics should empower students to "*communicate* appropriately using descriptions in *words*, *graphs, symbols, tables and diagrams"* (DoE 2011). We view LP as the relevant section of mathematics to accomplish the above objective. The intentions of LP are well described in its definition. In the Oxford Mathematics Study Dictionary, Tapson (1999) describes LP as: A method used to find the best solution to problems which can be expressed in terms of linear equations or inequalities. Solutions are usually found by drawing graphs of inequalities and looking for optimum values that satisfy the required conditions. This method is widely used in business and industrial contexts, and the problems often relate to obtaining maximum profits for given costs and production levels (Tapson 1999, 135).

The above definition also concurs with the Curriculum and Assessment Policy Statement 10–12 (DoE 2011), which is aimed at creating a life-long learner who is confident and independent, literate, numerate and multi-skilled, compassionate, has respect for the environment and can participate in society as a critical and active citizen. Problems in LP predominantly deal with concepts, symbols and graphs, which exposes students to opportunities to learn various forms of mathematical communication. LP brings together the algebraic, numeric and geometric or graphic representations that are fundamental in mathematics. The graphical representations in mathematics are often used to determine minimum costs or maximum production, which are applicable in industries and economics.

LP tasks are highly contextualised, and presented mostly in the form of word problems. Here is where the challenge comes in: problems are largely set in words or linguistic form, yet the solution demands students to invoke more than one form of representation (words, symbols and/or graphs). For multilingual students who do not learn mathematics in their main or home language, such problems require from them the competency in both the LoLT and the mathematical language, but also fluency in mathematical representation, i.e. symbolic, algebraic and graphical.

Reason for focusing on multilingual FET classrooms

Challenges relating to language and learning in bilingual and multilingual classrooms have long been recognised (Phakeng et al. 2018; Wagner and Moschovich 2018). Researchers in this area of study agree that learners' lack of or limited fluency in the LoLT puts additional demands on both lecturers and students. I refer to these as additional demands because they are not just about improving the students' fluency and accuracy in a language, which is what language teachers are concerned about. As Adler (2001) argue, on the one hand, mathematics educators have the dual task of teaching both mathematics and English at the same time. Students, on the other hand, have to cope with the new language of mathematics (Mathematical Language), the mathematical concepts (the mathematics), as well as the new language in which mathematics is taught (English). Presenting these multilingual students with LP tasks in the form of word problems, presents the opportunity to study how these linguistic competencies and fluencies come into play in mathematics learning. How the lecturer approach and solve LP problems during teaching in class may have a bearing on how students solve similar tasks.

Students' participation in the learning of mathematics

Recent curriculum reform in South Africa, and all over the world, reflects a change in what counts as mathematical learning (DoE 2011). The subject guidelines for mathematics (DoE 2011) claim that the learning of mathematics is not just about the learning of procedures or methods of solving particular problems, but students are also expected to explain the methods that they use. The students must also expect to discuss some of the procedures used to solve particular problems with their peers. As Moschkovich (2018) puts it, “students are now expected to participate in both verbal and written practices, such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments”. Moschkovich (2018) maintains that engaging in classroom mathematical discourse practices can be viewed broadly as talking and acting in the ways that mathematically competent people talk and act when debating mathematics in a classroom. These processes involve going beyond the use of technical terms. For students to be viewed as participating in mathematical classroom discourse, they need to be engaged in purposeful written exercises, talks or interactions when involved in mathematical activities (Moschkovich 2018). Such activities involve more than the routine use of technical or mathematical concepts. It is through such classroom activities that lecturers can assess if students understand the concepts taught and determine the support needed.

It is widely accepted that student participation is important for the meaningful learning of mathematics. What is continuously under debate is how educators can encourage the kind of participation that will lead to meaningful learning and also what types of student participation facilitate meaningful learning. Sfard et al. (1998) have deliberated the issue of whether learning mathematics through participation is as good as it is said to be. In their discussion, they refer to learning mathematics through conversation. I have interpreted that also to mean learning mathematics through participation since conversations are a product of participation. Sfard et al. (1998) argue that there are three different arguments for why participation is important for mathematics teaching and learning: the cognitivist argument, the interactionist argument, and the neo-pragmatist argument.

The cognitive argument holds that mathematical discussions or conversations enhance learners’ mathematical thinking. For instance, when students are involved in purposeful mathematical conversation among themselves, they learn how to explain the methods that they use to solve mathematical problems. The interactionist argument holds that the learning process is interactive since it is a process where educators and learners exchange ideas and learn from others. It therefore suggests that purposeful discussions that happen in mathematical classrooms encourage learners to share ideas and learn from each other. The neo-pragmatist argument states that knowledge can be expressed through discussions. This suggests that through discussions, learners can express what they know about certain mathematical concepts that they have previously learnt.

Is participation enough?

While various authors present different arguments about the importance of participation in mathematics teaching and learning, they all agree that it is not enough just to have learner participation in mathematics classroom activities and discourse. Participation in itself does not enable meaningful and productive learning. Mason (in Sfard et al. 1998) argues that for participation to be productive, it must be purposeful, it must be based on a clear mathematical topic, and learners must be genuinely involved and interactive. Lecturers play a role in initiating and promoting productive participation in the classroom. Hiebert et al. (1997) argue that lecturers should provide direction for mathematical activities and should plan goal-directed activities. If participation is critical to the learning process, learning activities should be planned in such a way that they allow productive participation to take place. This suggests that participation in mathematics classrooms depends on the mathematical activities in which the lecturers engage their students. As Cobb (in Sfard et al. 1998) suggests, the question is not whether students should participate in mathematical activities, but instead what the nature of that participation is that constitutes productive situations for mathematical learning.

Ensuring that participation is based on an explicit mathematical topic highlights the importance of classroom activities, the tasks that the lecturer gives and the questions they ask students. Stein et al. (2000) categorise mathematics tasks into two broad categories: "Lower-level and higher-level demand tasks. Lower-level demand tasks are those that only require learners to reproduce previously learned facts such as formulae, rules or definitions." These tasks require no explanation of the procedure that was used to solve a particular problem. These tasks have no potential to engage students in mathematical discussion since they demand less from the students (only requiring memorisation). Higher-level demands tasks are those that require cognitive effort from the students because the procedures for solving such tasks are usually implicit, and students are required to justify the methods that they use. These tasks have the potential to engage students in rich mathematical discussion since they require more from them because they are expected, for instance, to justify their conjectures. Of relevance to this study is how the selected task can facilitate students' participation in mathematical discussions in the learning of LP.

Stein et al. (2000) argue that how the lecturer implements the task is critical. The task in itself may not encourage student participation, but the kinds of probing questions that the lecturer asks are also important for the facilitation of participation. Lecturers play a role in initiating and channeling purposeful mathematical participation in a classroom. Hence, the focus of this study was also on what lecturers do to support multilingual students to solve LP problems.

It is important to note that the implementation of the tasks may not occur in the same way in a multilingual class in which students learn mathematics in a language that is not their home language. As indicated earlier on, this study will be undertaken in such a classroom. In this classroom, the students were still learning English while they learnt mathematics in English. It is, therefore, important to explore whether and how different language practices are used to encourage and support students' participation in mathematical learning. Moschkovich (1999) considered this concern by exploring how teachers support English language learners in learning mathematics and not only English. There is, therefore, a need to undertake a study like the one conducted by Moschkovich in a South African context and in FET colleges where the majority of students learn mathematics in a language that is not their mother tongue.

THEORETICAL FRAMEWORK

The situated perspective

Situated cognition views learning as participation in a community of practice (Lave 1996; Lave and Wenger 1991). They regard a community of practice as people who work together to accomplish goals, i.e. such people share common goals and motivation. The situated perspective maintains that a student learns mathematics through participation in a mathematical community and its practices. Lave and Wenger (1991, as cited in Lave 1996) define learning as "an aspect of changing participation in evolving communities of practice everywhere". They maintain that whenever people engage for periods, day by day, in practices in which their ongoing activities are interdependent, learning is part of their changing participation in the evolving practice. For this study, we regarded a mathematics classroom where there are students and their lecturer as a community of practice.

The above argument suggests that in a learning process, or rather in a learning community, ideas are co-produced through interaction, discussion and participation. Hanks (1991) maintains that an individual student can learn by engaging in a process. It, therefore, suggests that a student who enters a particular community of practice does not passively receive ideas from the group; he or she has to be a co-participant in the group. The learning of mathematics as participation in the community of practice, as advocated by the Theory of Situated Cognition, depends on the authentic activities that lecturers provide for students. It also depends on the mathematical practices that lecturers enforce in their classrooms. What this suggests is that if lecturers want meaningful learning to take place in their mathematics classrooms, they need to develop activities that allow this to take place.

Situated cognition regards a situation where learning takes place as equally important as learning itself. This perspective holds that the meanings that students construct are shaped by the situations or contexts in which they find themselves (Lave and Wenger 1991). This suggests that students construct meanings from their social talks or social situations, and bring such meanings to the mathematical classroom, particularly where they learn mathematics in a language that is not their home, main or first language. Lave and Wenger (1991) regard the learning of mathematics as participation in a community of practice where learners use language to communicate their mathematical ideas. Moschkovich (2018) is of the view that if one has to understand the conversation in classrooms where students learn mathematics in a language that is not their own, one needs to consider more than a few aspects of the situation. The social context, the languages that the learners speak and learners' historical context must also be taken into consideration. In a learning situation, this means that learners might bring meaning that is situationally constructed into the classroom, meaning that might not be part of correct mathematical discourses, and that could include mathematical practices. The teacher's role in this regard will be to elicit the correct mathematical meanings that learners are trying to convey. In this way, the learners' main language will be seen as a resource for learning and participating in mathematical activities, as opposed to being an obstacle to learning.

The situated perspective broadens the scope of looking into the students' participation in mathematical activities since it draws in more aspects about the situation of the learners, such as the resources that students use to construct meanings and how students construct such meanings. Where students lack words to express their mathematical ideas, they may draw on other resources such as the use of gestures and objects (Moschkovich 2018). This view is echoed in the curriculum reform in South Africa, which maintains that mathematics students should be able to, besides solving mathematical problems, communicate appropriately using words, symbols, tables graphs and diagrams (DBE 2011). As indicated earlier, in the South African context, students are allocated lecturers who should help them develop these skills through teaching. Teaching can take different forms and, among others, can include organising instruction, creating learning opportunities through teacher-led discussions, group work, class presentations and demonstrations. It also includes the use of different techniques for solving various mathematical problems located in different contexts.

This study is guided by the Theory of Situated Cognition of Learning and is informed by the assumption that how students participate, approach or solve mathematical activities is shaped by how they have been taught or what they have learned in the community of practice. This theory further suggests that students acquire skills through participation in authentic contexts and by communicating with peers and experts (lecturers) about these contexts (Lave 1996; Lave and Wenger 1991). Central to this theory is the principle of Legitimate Peripheral Participation (LPP) (Hanks 1991), which advocates that an individual student acquires skills to perform by engaging in the process under the attenuated conditions of LPP where the central idea is that a student participates in the actual practice under the guidance of an expert. In LPP, a novice or newcomer (students) is transformed to become skilled by the master (lecturer) through engaging in the process. The end-product of LPP is "when students come to reproduce aspects of the performance style of a charismatic teacher" (Hanks 1991, 19). This suggests that the strategies that students use to solve mathematical tasks are likely a reflection of how lecturers have solved similar tasks in class.

Singley and Anderson (as cited in Anderson, Reder and Simon 1996) argue that "the transfer between tasks is a function of the degree to which the tasks share cognitive elements". In this study, an LP task was given to students to solve. In LP, there are commonly used basic concepts (for example, "at least", "at most", and so on) and these concepts have been included in this task. This task was, therefore, at the same cognitive level as the tasks treated in the teaching of the topic in class. When solving the written task in the study, students were expected to transfer the knowledge that they gained during the lesson when solving similar problems or tasks.

Anderson et al. (1996) argue that in situated learning, there can be either large amounts of transfer, a modest amount of transfer or no transfer of skills at all. The above argument suggests that in any teaching and learning situation, not all of the students will understand all of the concepts and procedures and be able to transfer that knowledge to other situations. In this study, following from the above arguments, the students had different levels of understanding of the concepts and strategies used when solving tasks in class. This led them to perform differently in the written tasks that they were given during the data collection.

The situated perspective was deemed the most appropriate model to support the discontinuity model in analysing conversations and practices in classrooms where students learn mathematics in a language that is not their home, main or first language. The next section presents the literature that was reviewed for this study.

RESEARCH DESIGN AND METHODOLOGY

In this section, we discuss the research design and methodology that was used to collect and analyse the data. It also presents the ethical issues that were considered during the study. This was a qualitative case study involving the NC(V) Level 3 Mathematics class where students do mathematics in a language that is not their first, home or main language. The use of a qualitative case study affords the researcher opportunities to collect extensive data on the individual(s), programme(s), or event (s) on which the investigation focused. Neuman (1997) states that qualitative researchers use a case study approach to gather a large amount of information on one or a few cases, gain deeper insight, and get more details on the cases being examined.

The selection of the case for these students and their lecturer with their linguistic background afforded us opportunities to explore the strategies that they used when working on LP tasks, as well as the language practices that they utilised. Although the main focus was on students' approaches to LP tasks, it is impossible to exclude lecturers due to the interdependency of teaching and learning in the South African context. In the teaching and learning of any subject, including mathematics, the lecturer has the primary responsibility of teaching students conventional strategies for solving mathematical tasks.

Research participants

This study involved students and their lecturer from one campus at an FET College in Gauteng. The campus is situated in a Central Business District (CBD) in Gauteng and has the potential of admitting students from different provinces in the country. Therefore, the students come from a different language, social and cultural backgrounds. The campus had only 18 Level 3 students doing mathematics, and 16 students completed the task. The lecturer was selected to satisfy the research criteria of a multilingual lecturer teaching mathematics Level 3. Like his students, the lecturer also had to be multilingual and teaching mathematics in a language (English) that was not his main, home or first language.

Data collection

The following sources of data were used in this study: students' written tasks, students' interviews, as well as a reflective interview with their lecturer. The lecturer's interview was based on the students' responses to the written task and their reflective interviews on how and what prompted them to respond to the task how they did. The researcher provided a written task on LP, administered by the lecturer in the presence of the researcher. The written task was marked, and the students' performance was analysed by the researcher. Two students were selected purposely for reflective interviews based on their performance and the level of conceptualisation shown when responding to the task. These were the students who obtained the highest and lowest scores on the written task. A reflective interview was held with the lecturer to probe his teaching and support strategies implemented when teaching the concepts of LP. All interviews were audio-recorded and transcribed. The lecturer interview was semi-structured and provided data on the lecturer's intentions regarding the lessons, and interpretations of how he taught the concepts during the lessons. During the reflective interview, the lecturer's view was probed on the success of his lessons and the role that his language practices played in students' ways of solving the task.

The data from this interview provided insights into the lecturer's reasons for assisting students in the way he did, and why he thought it supported students' mathematical learning.

Data Analysis

- Data analysis is a systematic search for meaning (Hatch 2002). Data analysis is the stage where the researcher begins to answer the research questions to communicate the findings of the research. Creswell (1998) and Stake (1995) (both as cited in Leedy and Ormrod 2004) argue that data analysis in case studies typically involves the following stages:
- Organisation of details about the case: The stage where facts about the case are arranged in a logical order.
- Categorisation of data: The stage where categories or clustering of data into meaningful groups is done.
- Interpretation of single instances: The stage where specific occurrences are examined for specific meanings that they might have concerning the case.
- Identification of patterns: The stage where data and their interpretations are scrutinised more broadly than a single piece of information.
- Synthesis and generalisations: An overall image/portrait of the case is constructed.

In this study, we did an inductive analysis as described by Hatch (2002), who views it as a kind of analysis where the categories for data analysis are developed as one analyses the data. This implies that in this study, categories were developed based on what emerged from the students' responses to the written task and from the reflective interviews with both the students and their lecturer.

Analysis of the students' responses to the written task

Sixteen students were given a written task on LP without time restriction, but to be completed on the same day. The students' responses were marked and analysed by the researcher.

Analysis of the written task

At the end of each academic year, in NC(V) levels 2 – 4, students write national examinations that are set and moderated externally. From each examination paper, we selected one question which assessed students' knowledge of LP.

An LP task from the exemplar question paper published in June 2008 by the national DoE (2008) was analysed before it was administered to the students. The rationale for analysing the task first was to indicate the levels of cognitive demand that are expected from the students to successfully respond to the task. Analysing the task first alleviated some possible concerns about the nature or the kind of the tasks administered to the students during the study.

Exemplar question papers are mirror-images of the real examination question paper. They are aimed at reflecting and alerting the lecturers to the structure and standard of the final examination question paper. The task presented below is a typical example since the nature of questioning and the concepts used are the same.

Exemplar Question paper (June 2008)

1 JJ Blocks, a block manufacturing company, is intending to **launch** a new and **improved** building block. For the manufacturing process, a mixture of two sand types A and B is required. At least 800kg, but not more than 1200kg of the two sand types are needed to prepare a batch of the new blocks. The mixture must contain at least 2kg of A to every 1kg of B, and at least 200kg of B must be used in the batch.

1.1 Present this information as a system of inequalities. (4)

1.2 Graph the system and indicate the feasible region. (5)

1.3 Use the graph to find the most economical mixture if sand type A costs R4/kg and sand type B costs R7/kg. Indicate the search line on the graph. (5)

1.4 Determine the minimum cost required. (2)

(DoE June 2008)

FINDINGS AND DISCUSSIONS

Students' attitude towards word problems and LP

Questions on LP are mostly presented in the form of word problems. The students' attitude towards word problems is likely to be the same as their attitude towards LP. The subject guidelines (DoE 2011) for mathematics Levels 2–4 maintain that mathematics should empower students to communicate appropriately using descriptions in words, graphs, symbols, tables and diagrams. This is what LP is all about. It deals with translating statements in words to symbolic form, representing constraints graphically, amongst others.

Research on the impact of students' attitude towards mathematics in general, word problems, and LP, in particular, has been a hot topic for a while now (Arizpe, Dwyer, and Stevens 2009; Ashby 2009; Schackow and Thompson 2005). Ashby (2009) argues that mathematics is a subject that requires a considerable amount of perseverance and determination from individuals to succeed. He further claims that a negative attitude towards mathematics could considerably reduce a person's willingness to give their maximum performance in conducting a task. This suggests that without the ability to persevere, mathematical development is likely to be difficult.

It is, therefore, up to educators to prepare their students mentally so that they can approach word problems and mathematics in general with perseverance and determination. In the reflective interview with the lecturer in this study, when he was asked how he went about introducing the topic of LP to his students, this is what he had to say:

> Researcher: "If you may share with me, how did you introduce this topic?"
>
> Lecturer: "I started by explaining to students where this section of maths is applied, and then I explain[ed] the linear programming terms."

The above extract shows that the lecturer tried to get students to see some connections between this branch of mathematics, LP, and its application in the real world. This helps students to produce acceptable solutions since they should be able to attach meaning to what is happening in the problem. In other words, the lecturer prepared the students to be able to see the relationships and connections between the quantities provided by the LP problem and their real-life knowledge. The approach above helps to change students' attitude positively before starting with the LP concepts that are presented in the form of word problems.

The above approach used by the lecturer is pro-active in preventing any negative attitude that may prevail in terms of the topic before starting to solve the problems presented in the topic. Ashby (2009) states that, "Children may find the nature of linear programming difficult to cope with as its wider-reaching implications can be hard to see" (Ashby 2009, 7). He further claims that it is surprising that pupils across the entire ability range are unable to make connections between LP and its much more practical use, hence the negative attitude. This claim suggests that from his focus group, children found it difficult to make a connection between the work they do in class and its practical applications in their daily life experience. Children cannot be expected to make these connections without the assistance of an educator who should motivate, show connections and help to create the right attitude. Askew et al. (in Ashby 2009) argue that the most effective educators are connectionists. These are the educators who cause students to see the value and relevance of mathematical concepts in real life. Understanding the purpose of mathematics and LP, in particular, will not only help to improve motivation and attitude towards it but could also help in the actual understanding of the concepts presented in the topic.

Schackow and Thompson (2005) argue that how students view mathematics, as well as their attitude towards it, especially word problems, has an impact on their success. In their analysis of the Third International Mathematics and Science Study (TIMSS), McCreith and Lapointe, (2005, as cited in Schackow and Thompson 2005) found that the strongest predictor of participation in an advanced mathematics course was students' attitude towards mathematics in general. This finding suggests that educators should play a motivating role so that students can develop a positive attitude towards mathematics as a subject and its sub-fields, including word problems and LP. Their finding on students' attitude concurs with the results of this study. In terms of this, when Taki (pseudonym) was asked how she experienced the task, this is how she responded:

Researcher:	"Shoo shoo [language used to show appreciation] I just want us to go through very few questions and I don't think it will take long. As you were doing the task how did you feel?"
Taki:	"Eeh, the task was difficult."
Researcher:	"Why do you say the task was difficult?"
Taki:	"Is because I do not understand linear programming."
Researcher:	"Why, why do you say you do not understand linear programming? Exactly what is it that you do not understand in linear programming?"
Taki:	"I get confused with the signs, like when they say 'at least' and 'at most' and one other thing, there are word problems and *nna a ke otlwani le tsona*" [I don't like them].
Researcher:	"Why *o sa otlwani le tsona*" [why don't you like them]?
Taki:	"*eish, fela*" [just].

In the above conversation, it was clear that the student had a negative attitude towards word problems, and LP is mostly communicated through word problems. Because of her attitude, Taki may never focus fully on this sub-field of mathematics. From my own teaching experience, students with a negative attitude towards any schoolwork tend to lose confidence in their work. Without confidence, students tend to underestimate their potential, and this is what causes failure because they avoid challenges. They do not believe that they can solve difficult problems. Olson (1998, as cited in Schackow and Thompson 2005) argue that students believe that word problems are difficult, and they become frustrated since these problems are typically not solved quickly. This belief and lack of confidence are likely to influence students' attitude, problem-solving strategy and perseverance in solving multi-step problems. Educators should, therefore, use different teaching methods and multiple representations to awaken students' interest in this section of mathematics; it may also help in creating a positive attitude.

Co-operative group work enhances understanding

According to Calderón, Hertz-Larowitz, and Slavin (1998), co-operative group work is "when two or more individuals who routinely function as a team are interdependent in achievement of a common goal". Group work is a teaching technique that facilitates learning, particularly if it is well managed by the educator. In the teaching and learning of mathematics, group work is a technique that affords students an opportunity to learn from each other. There are various types of group work that can be employed by educators in the teaching and learning of mathematics (Calderón et al. 1998). They give a distinction between two types of group work, namely, "collaborative learning" and "co-operative group work". In collaborative learning, educators compose heterogeneous groups, and individuals in groups are assigned responsibilities and less active monitoring by an educator is required. In co-operative group work, there is often a voluntary group formation, and individuals in groups are responsible for their learning with group support where teaching and learning is a shared experience between an educator and their students. In this study, the technique implemented was co-operative group work since the lecturer's role was to observe, coach and interact with small groups (Calderón et al. 1998). During the teaching and learning of this topic (LP), students just grouped themselves according to their social relations. This is what one of the students said about their group formation:

Researcher:	"Okay! Good! Good! Eh! Now, how did you get to know about doing this? Tell me how did you come to know about doing this task this way?"
Tshepo:	"The lecturer taught us."
Researcher:	"Besides the lecturer, could you have done it any other way?"
Tshepo:	"Yes, by going back to our study groups that we have. Having different students from different campuses to discuss it in different languages, again like I mentioned earlier. So, that's how we did it."
Researcher:	"Oh! Maybe like for example, eh! Students who come from different campuses might have been taught differently?"
Tshhepo:	"Yes."
Researcher:	"Okay! I see."
Tshepo:	"Or decided to share what we have together and combined our knowledge and it became so much easier for us."
Researcher:	"Oh! Tell me about your groups. How are they set up?"
Tshepo:	"No, it is our permanently set students studying most the subjects, especially maths together. We just group ourselves *ka se chomi nje*!" [on friendship basis]

The above comments from Tshepo show that the lecturer never played a role in the grouping of the students. The students simply grouped themselves, just like she put it: "We just group ourselves *ka se chomi nje*!" [on friendship basis].

People who are in a friendship relate well in some aspects of life and share common interest. In this class this can be students coming from the same campus, same township and same high school or sharing same language. Such students have a potential of interacting freely with one another in the group. During group discussion they can openly share their opinions, ideas and ways of doing things.

When the lecturer was asked about how he went about grouping the students, this is what he said:

> Researcher: "How did you group them?"
>
> Lecturer: "I found them grouped or actually they grouped themselves, and surprisingly, most of the groups were ethnically grouped. We find that these particular students who speaks same language will be in the same group and it makes it easier for them to explain to each other because they will be able to understand better when they are explained in their home language."
>
> Researcher: "Do you mean these students are not grouped according to their mathematical capabilities?"
>
> Lecturer: "No, they have been grouped according to their ethnic groups and not according to their mathematical abilities."

The above statement made by the lecturer affirms that the type of grouping in his class was co-operative since the students voluntarily grouped themselves. In fact, this improves students' opportunity to communicate and express their ideas freely in small groups, and it enhances students' ability to reflect on their assumptions and thinking processes. The first author of this article has 17 years of teaching experience, which taught him that belonging to a small and supportive group, and participating in small-group activities has the potential of developing students' higher-order thinking skills, which will enhance the flexible use of their knowledge and, as a result, will improve students' success. In groups, students discuss their mathematical ideas freely.

Orr (as cited in Moore 2002) argues that "classroom discourse enables educators to identify, diagnose and address the problems and misconceptions that students possess". As it was the case in the study, the lecturer had been observing and interacting with the small groups in the class, and it was through this process that students' problems and misconceptions could be identified and addressed. When asked about what happened in her group, Taki said:

> "*Ga re sa thlaloganyi re a mobitsa* [when we do not understand we call him] we raise up our hands and then *o a tla* [he comes] and then he answers us and sometimes he uses our language to answer our question." (Taki)

The above statement made by Taki affirms the role of the lecturer in co-operative group work. In fact, in utilising this technique, the lecturer plays the role of a coach – he gives instructions, guides and corrects where there are deviations. It is through this process that he can identify students with misconceptions and uproot these misconceptions before they become permanent.

Group discussion is not only vital in developing students' communication and reasoning skills but enhances the lecturer's ability to evaluate the students' progress and to analyse their effectiveness in their groups. In addition, such a discourse also allows students to experience the process of attaining mathematical understanding through their peers, other than the lecturer, and this augments their mathematical empowerment. Lecturers should create a learning environment that fosters the development of each student's mathematical ability by respecting and valuing students' ideas, way of thinking, and mathematical dispositions, and encouraging students to work co-operatively to make sense of mathematics (Moore 2002). To substantiate her statement, Moore uses an instructional method called "Ask Three Before You Ask Me". In this method, groups work together on problems, and they are expected to consult one another before they take their enquiry to the educator. This method could be very helpful because some students become more comfortable in their small study groups and can thus learn better. In the reflective interview, the two students perceived group work to be beneficial and found that it promotes communication and increases their mathematical understanding.

In the extract quoted earlier on, Tshepo said "... or decided to share what we have together and combined our knowledge and it became so much easier for us". Her statement suggests that during group discussions, students share ideas and ways of doing things and it enhances their understanding. Her view concurs with Moore's (2002) argument, which, on the one hand, maintains that if you bring two or more people together, then you might discover better and alternative ways of how to do things. On the other hand, Taki understands the mathematical concepts when they discuss these in class, but when she is alone, activities containing similar tasks become difficult for her. This is how she put it in the reflective interview:

Researcher:	"Any other challenges regarding the task?"
Taki:	"Mmh, when we do this task as a group I understand and we get them right but when I am alone it becomes difficult."
Researcher:	"What becomes difficult?"
Taki:	"I don't understand how to develop constrains when I'm at home."
Reseacher:	"But when you discuss this in your groups you are able to develop constrains right?"
Taki:	"Jah at least we help each other, and we use our language."

The above statements suggest that Taki has a problem understanding mathematical concepts on her own, but during group discussions she was able to understand the content. This is evidence that the teaching technique, co-operative group work, opens up a platform for the enhancement of mathematical conceptual understanding, hence Taki claimed she understood better when working in her group. It, therefore, remains with the lecturer to nurture the appropriate attitude of the students towards the concepts at hand, and the effectiveness of the co-operative group in completing their work. If not well monitored, it could be just fun and meaningless, which may then mean that the work (and the group) loses its focus and value.

Both Tshepo and Taki attested to the fact that group discussions helped them to understand concepts better. In the written task, Tshepo performed very well, whereas Taki performed very poorly. Taki claimed that during group discussions she understood and could get the correct answer but could not do the task on her own.

CONCLUSION

It is through language that lecturers interact with students in carrying out mathematics activities. In this study, the LoLT was English, which was not the students' first language. The data analysed showed that the students had low language proficiency in the LoLT. This then calls for lecturers to apply different language practices that will enable students to understand the language in which the tasks are presented. It is from understanding the task that students can employ different solution strategies for solving the tasks presented to them.

The data analysed in this study revealed that students' lack of proficiency in the LoLT hindered their understanding of the task, and this restricted them from developing correct constraints in the task. As a solution to this challenge, the lecturer practised code-switching, which was practised as a teaching strategy to ensure an understanding of LP concepts in the lecturer's classroom. The lecturer in this study mostly used co-operative group work where students engaged in group discussions and learnt from one another. The students became more actively engaged in mathematical problem solving through co-operative learning. During their small group discussions, the students were deliberately allowed to use their home languages to discuss LP concepts. The use of students' home languages enhanced their conceptual understanding. The students were allowed to use their home languages to discuss concepts in their different groups, to explain concepts to the whole class in their languages and after that solve the task. According to the students, these practices enhanced their understanding of basic LP concepts that are commonly used in LP problems, and it further developed their mathematical vocabulary. Through an understanding of the concepts, as a result of the use of their home languages, the students were able to develop constraints for the restrictions of LP problems, i.e. they were able to translate mathematical statements into symbolic form. Code-switching and discussions in their home languages enabled them to understand the procedures and strategies involved in the solving of LP problems.

REFERENCES

Adler, J. 2001. *Teaching and learning mathematics in multilingual classrooms*. Dordrecht, Netherlands: Kluwer Academic Publishers.

Anderson, J. R., L. M. Reder, and H. A. Simon. 1996. "Situated learning and education." *Educational Researcher* 25(4): 5–11.

Arizpe, O., J. Dwyer, and T. Stevens. 2009. "Mathematical Self-Efficacy of Middle School Students Solving the Rubik Cube." *International Journal for Mathematics Teaching and Learning* 3(2009): 1-11. https://www.cimt.org.uk/journal/dwyer2.pdf.

Ashby, B. 2009. "Exploring Children' Attitude towards mathematics." *Proceedings of the British Society for Research into Learning Mathematics* 29(1): 7–12.

Barwell, R. 2018. "From language as a resource to sources of meaning in multilingual mathematics classrooms." *The Journal of Mathematical Behavior* 50: 155–168.

Calderón, M., R. Hertz-Larowitz, and R. Slavin. 1998. "Effects of bilingual co-operative integrated reading and composition on students making the transition from Spanish to English reading." *The Elementary School Journal* 99(2): 153–165.

DBE *see* Department of Basic Education.

Department of Education. 2008. *National Certificate (Vocational) Examinations and Assessments: Mathematics Level 3*. Pretoria: Department of Education.

Department of Basic Education. 2011. *Curriculum and Assessment Policy Statement 10–12 (General): Mathematical Literacy.* Pretoria: Department of Education.

Department of Education. 2011. *National Certificate (Vocational) (NCV), Subject Guidelines: Mathematics NQF Level 3.* Pretoria: Department of Education

DoE *see* Department of Education.

Hanks, F. 1991. "Foreword by William F. Hanks." In *Situated learning: Legitimate peripheral participation*, ed. J. Lave and E. Wenger, 13–24. Cambridge: Cambridge University Press.

Hatch, J. A. 2002. *Doing qualitative research in education settings*. Suny Press.

Hiebert, J., T. P. Carpenter, E. Fennema, K. C. Fuson, D. Wearne, H. Murray, A. Olivier, and P. Human. 1997. *Making sense: Teaching and Learning Mathematics with Understanding.* Portsmouth: Heinemann.

Lave, J. 1996. "Teaching, as Learning, in Practice." *Mind, Culture, and Activity* 3: 149–164.

Lave, J. and E. Wenger. 1991. *Situated learning: Legitimate peripheral participation*. New York, NY: Cambridge University Press.

Leedy, P. D. and J. E. Ormrod. 2004. *Practical Research*. Upper Saddle River, New Jersey Columbus, Ohio: MERRILL Prentice Hall.

Moore, J. 2002. African American male Students' Perception of a mathematics Learning Environment. *Academic Exchange Quarterly Fall* 6(3): 1096–1453.

Moschkovich, J. 2018. "Talking to learn mathematics with understanding: Supporting academic literacy in mathematics for English learners." In *Language, Literacy, and Learning in the STEM Disciplines*, 13–34. Abingdon, Oxfordshire, UK: Routledge.

Neuman, W. L. 1997. *Social research methods: Qualitative and quantitative approaches*. Boston, MA: Allyn and Bacon.

Phakeng, M. 2018. *One country, many languages: Exploring a multilingual approach to mathematics teaching and learning in South Africa*. https://hal.archives-ouvertes.fr/hal-01849650/.

Phakeng, M. S. and J. N. Moschkovich. 2013. "Mathematics education and language diversity: A dialogue across settings." *Journal for Research in Mathematics Education* 44(1): 119–128.

Phakeng, M. S., N. Planas, A. Bose, and E. Njurai. 2018. "Teaching and learning mathematics in trilingual classrooms: Learning from three different continents." In *Mathematical discourse that breaks barriers and creates space for marginalised learners*, 277–293. Leiden, Netherelands: Brill Sense.

Planas, N. 2018. "Language as resource: A key notion for understanding the complexity of mathematics learning." *Educational Studies in Mathematics* 98(3): 215–229.

Schackow, J. B. and D. R. Thompson. 2005. "High school students' attitudes toward mathematics." *Academic Exchange Quarterly* 9(3): 12–19.

Sfard, A., P. Nesher., L. Streefland, P. Cobb, and J. Mason. 1998. "Learning mathematics through conversation: Is it as good as they say?" *For the Learning of Mathematics* 18(1): 41–51.

Shackow, B. and D. R. Thompson. 2005. *High School Students' Attitude Towards Mathematics*. http://findarticle.com/p/articles./mi-hb3325./is-3-9/ai-n29219648/.

Stein, M. K., M. S. Smith, M. A. Henningsen, and E. A. Silver. 2000. *Implementing standards-based mathematics instruction: A casebook for professional development.* New York, NY: Teachers' College Press.

Tapson, F. 1999. *Oxford Mathematics Study Dictionary*. New York, NY: Oxford University Press.

Wagner, D. and J. N. Moschkovich. 2018. "International perspectives on language and communication in mathematics education." In *Language and Communication in Mathematics Education*, 3–9. Springer.